

Density Histograms and the LLN

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

1. Repeated iid samples from a population with mean 18.9 produced sample means (add 8.9 to each listed value): n=10: 8.7,11.2,9.4,10.8; n=100: 9.8,10.2,10.1,9.9; n=1000: 10.02,9.98,10.01,9.99. Interpret using the LLN, and distinguish one original observation X from the average Xbar.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

2. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 10,16,15; total 41. Which bin has greatest density?

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Density Histograms and the LLN

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

3. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,12,18; total 39. Find each relative-frequency density.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

4. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,10,19; total 38. Find the relative frequency in [1,3) from bar area.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Density Histograms and the LLN

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

5. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,11,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

6. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 12,13,14; total 39. Which bin has greatest density?

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Density Histograms and the LLN

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

7. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 8,12,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

8. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,14,14; total 37. Find the relative frequency in [1,3) from bar area.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

## Density Histograms and the LLN

### Worked Answer Key

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

1. Repeated iid samples from a population with mean 18.9 produced sample means (add 8.9 to each listed value):  $n=10$ : 8.7,11.2,9.4,10.8;  $n=100$ : 9.8,10.2,10.1,9.9;  $n=1000$ : 10.02,9.98,10.01,9.99. Interpret using the LLN, and distinguish one original observation  $X$  from the average  $\bar{X}$ .

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

**Answer: Larger- $n$  sample means concentrate near 18.9; one  $X$  remains one population draw**

1. One original observation  $X$  is a single population draw; increasing  $n$  does not make that draw itself an average.
2.  $\bar{X}$  averages  $n$  iid observations, has variance  $\sigma^2/n$ , and the supplied means show decreasing spread around the population mean.
3. This illustrates LLN concentration, not a claim that the sampling shape is normal.

2. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 10,16,15; total 41. Which bin has greatest density?

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

**Answer: bin 1, density 0.2439**

1. Compare count/(total x width), not raw counts.
2. bin 1, density 0.2439

Density Histograms and the LLN

Worked Answer Key

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

3.

**Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,12,18; total 39. Find each relative-frequency density.**

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

**Answer: 0.2308, 0.1538, 0.1538**

1. density=count/(total x width).

2. 0.2308, 0.1538, 0.1538

4.

**Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,10,19; total 38. Find the relative frequency in [1,3) from bar area.**

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

**Answer: 0.2632**

1. area=count/total=10/38=0.2632 (equivalently width times the unrounded density).

## Density Histograms and the LLN

### Worked Answer Key

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

5. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,11,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

**Answer: density x width = relative frequency**

1. Density height is  $\text{count}/(\text{total} \times \text{width})$ ; every bar area is  $\text{count}/\text{total}$  and total area is 1.

6. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 12,13,14; total 39. Which bin has greatest density?

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

**Answer: bin 1, density 0.3077**

1. Compare  $\text{count}/(\text{total} \times \text{width})$ , not raw counts.
2. bin 1, density 0.3077

Density Histograms and the LLN

Worked Answer Key

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

7. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 8,12,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: density x width = relative frequency

1. Density height is count/(total x width); every bar area is count/total and total area is 1.

8. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,14,14; total 37. Find the relative frequency in [1,3) from bar area.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.  
Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: 0.3784

1. area=count/total=14/37=0.3784 (equivalently width times the unrounded density).