

Covariance of Random Variables

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

- 1. Let X be equally likely to be -11.3, 0, or 11.3, and define $Y=X^2$. Show $\text{Cov}(X,Y)=0$ and decide whether X and Y are independent.**

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

- 2. A joint PMF assigns probability $1/6$ to each pair (63,128), (69,144), (69,148), (75,152), (75,156), (81,172). Find $\text{Cov}(X,Y)$ using $E(XY)-E(X)E(Y)$.**

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Covariance of Random Variables

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

- 3. Paired observations (7 pairs): (56,60), (60,66), (58,66), (62,66), (60,66), (64,72), (62,66). Treat these as a sample. Find covariance using denominator $n-1$.**

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

- 4. Paired observations (5 pairs): (-74,-222), (-70,-208), (-70,-206), (-66,-198), (-66,-196). Treat these as the whole population. Find covariance using denominator n .**

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Covariance of Random Variables

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

5. Paired observations (5 pairs): (70,215), (73,225), (74,229), (77,236), (78,240). Treat these as the whole population. Find $\text{Var}(2X-Y)$.

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

6. Paired observations (8 pairs): (49,197), (53,218), (53,223), (57,229), (57,234), (61,255), (61,245), (65,266). Treat these as the whole population. Find $\text{Cov}(3X+2, 3Y+4)$.

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Name _____ Score _____ Teacher _____ Date _____

Covariance of Random Variables

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

7. Paired observations (5 pairs): (22,19), (27,27), (26,29), (31,28), (30,30). Treat these as the whole population. Find correlation.

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

8. Paired observations (6 pairs): (-17,-52), (-13,-36), (-15,-38), (-11,-34), (-13,-36), (-9,-20). Treat these as the whole population. Find covariance using denominator n .

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Covariance of Random Variables

Worked Answer Key

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

- 1. Let X be equally likely to be -11.3, 0, or 11.3, and define $Y=X^2$. Show $\text{Cov}(X,Y)=0$ and decide whether X and Y are independent.**

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: $\text{Cov}=0$; X and Y are not independent

1. By symmetry $E(X)=0$ and $E(XY)=E(X^3)=0$, so covariance is 0.
2. Y is determined by X , and $P(Y=0 | X=0)=1$ differs from $P(Y=0)=1/3$, so they are dependent.

- 2. A joint PMF assigns probability $1/6$ to each pair (63,128), (69,144), (69,148), (75,152), (75,156), (81,172). Find $\text{Cov}(X,Y)$ using $E(XY)-E(X)E(Y)$.**

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: 74.0000

1. $E(X)=72.0000$, $E(Y)=150.0000$, $E(XY)=10874.0000$.
2. $\text{Cov}(X,Y)=10874.0000-(72.0000)(150.0000)=74.0000$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Covariance of Random Variables

Worked Answer Key

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

3. Paired observations (7 pairs): (56,60), (60,66), (58,66), (62,66), (60,66), (64,72), (62,66). Treat these as a sample. Find covariance using denominator $n-1$.

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: 8.0000

1. Means are $\bar{x}=60.2857$, $\bar{y}=66.0000$.
2. Sum paired deviation products and divide by 6: covariance=8.0000.

4. Paired observations (5 pairs): (-74,-222), (-70,-208), (-70,-206), (-66,-198), (-66,-196). Treat these as the whole population. Find covariance using denominator n .

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: 27.2000

1. Means are $\bar{x}=-69.2000$, $\bar{y}=-206.0000$.
2. Sum paired deviation products and divide by 5: covariance=27.2000.

Covariance of Random Variables

Worked Answer Key

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

5. Paired observations (5 pairs): (70,215), (73,225), (74,229), (77,236), (78,240). Treat these as the whole population. Find $\text{Var}(2X-Y)$.

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: 9.3600

1. $\text{Var}(2X-Y) = 4\text{Var}(X) + \text{Var}(Y) - 4\text{Cov}(X, Y)$.
2. Substitution gives 9.3600.

6. Paired observations (8 pairs): (49,197), (53,218), (53,223), (57,229), (57,234), (61,255), (61,245), (65,266). Treat these as the whole population. Find $\text{Cov}(3X+2, 3Y+4)$.

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: 886.5000

1. Translations do not change covariance and scaling multiplies it: $\text{Cov}(aX+b, cY+d) = ac \text{Cov}(X, Y)$.
2. $\text{Cov}(X, Y) = 98.5000$, so $(3)(3)\text{Cov}(X, Y) = 886.5000$.

Covariance of Random Variables

Worked Answer Key

State whether data are a population or sample. Use the specified denominator, keep paired observations together, and distinguish zero covariance from independence.

7. Paired observations (5 pairs): (22,19), (27,27), (26,29), (31,28), (30,30). Treat these as the whole population. Find correlation.

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: 0.8208

1. $\text{Cov}=10.2800$, $\text{Var}(X)=10.1600$, $\text{Var}(Y)=15.4400$.

2. $r=\text{Cov}/\sqrt{\text{Var}(X)\text{Var}(Y)}=0.8208$.

8. Paired observations (6 pairs): (-17,-52), (-13,-36), (-15,-38), (-11,-34), (-13,-36), (-9,-20). Treat these as the whole population. Find covariance using denominator n .

Assumptions: The stated observations and denominator convention define the dataset; no pairs are omitted.

Paired products use matching observations. Population covariance divides by n ; sample covariance divides by $n-1$.

Answer: 22.6667

1. Means are $\bar{x}=-13.0000$, $\bar{y}=-36.0000$.

2. Sum paired deviation products and divide by 6: covariance=22.6667.