

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Continuous Random Variable Transformations

Write the transformed support, full piecewise CDF, and PDF. Account for decreasing maps and every valid preimage.

- 1. X is Uniform(0,17) and $Y=X^2$. Write the support and full piecewise CDF and PDF of Y.**

Reference: On nonnegative support, x maps monotonically to x^2 and the inverse is $\sqrt{y}$.

- 2. X is Uniform(0,18) and $Y=3X+0$. Write the support and full piecewise CDF and PDF of Y.**

Reference: For a monotone map, solve the inequality for X . A negative slope reverses it; the density divides by the absolute derivative.

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Write the transformed support, full piecewise CDF, and PDF. Account for decreasing maps and every valid preimage.

3. X is Uniform(0,2) and $Y=-4X+1$. Write the support and full piecewise CDF and PDF of Y .

Reference: For a monotone map, solve the inequality for X . A negative slope reverses it; the density divides by the absolute derivative.

4. X is Uniform(-3,3) and $Y=X^2$. Write the support and full piecewise CDF and PDF of Y , accounting for both preimages.

Reference: For $0 < y < \text{upper}^2$, the event $X^2 \leq y$ is $-\sqrt{y} \leq X \leq \sqrt{y}$; both inverse branches contribute to the density.

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5. X is Uniform(0,16) and $Y=\sqrt{X}$. Write the support and full piecewise CDF and PDF of Y.

Reference: The inverse is $x=y^2$. Differentiate $F_X(y^2)$ on the transformed support.

6. X is Uniform(0,25) and $Y=\sqrt{X}$. Write the support and full piecewise CDF and PDF of Y.

Reference: The inverse is $x=y^2$. Differentiate $F_X(y^2)$ on the transformed support.

Continuous Random Variable Transformations

Write the transformed support, full piecewise CDF, and PDF. Account for decreasing maps and every valid preimage.

7. X is Uniform(-6,6) and $Y=X^2$. Write the support and full piecewise CDF and PDF of Y, accounting for both preimages.

Reference: For $0 < y < \text{upper}^2$, the event $X^2 \leq y$ is $-\sqrt{y} \leq X \leq \sqrt{y}$; both inverse branches contribute to the density.

8. X is Uniform(0,7) and $Y=-3X+1$. Write the support and full piecewise CDF and PDF of Y.

Reference: For a monotone map, solve the inequality for X. A negative slope reverses it; the density divides by the absolute derivative.

Continuous Random Variable Transformations

Worked Answer Key

Write the transformed support, full piecewise CDF, and PDF. Account for decreasing maps and every valid preimage.

- 1. X is Uniform(0,17) and $Y=X^2$. Write the support and full piecewise CDF and PDF of Y.**

Reference: On nonnegative support, x maps monotonically to x^2 and the inverse is $\sqrt{x}$.

Answer: support [0,289]

1. $F_Y(y)=0$ for $y<0$; $F_Y(y)=\sqrt{y}/17$ for $0\leq y\leq 289$; $F_Y(y)=1$ above 289.
2. $f_Y(y)=1/(2(17)\sqrt{y})$ for $0<y<289$, and 0 otherwise.

- 2. X is Uniform(0,18) and $Y=3X+0$. Write the support and full piecewise CDF and PDF of Y.**

Reference: For a monotone map, solve the inequality for X. A negative slope reverses it; the density divides by the absolute derivative.

Answer: support [0,54]

1. $F_Y(y)=0$ for $y<0$; $F_Y(y)=(y-0)/54$ for $0\leq y\leq 54$; $F_Y(y)=1$ for $y>54$.
2. $f_Y(y)=1/54$ for $0<y<54$, and 0 otherwise.

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Worked Answer Key

Write the transformed support, full piecewise CDF, and PDF. Account for decreasing maps and every valid preimage.

3. X is Uniform(0,2) and $Y=-4X+1$. Write the support and full piecewise CDF and PDF of Y .

Reference: For a monotone map, solve the inequality for X . A negative slope reverses it; the density divides by the absolute derivative.

Answer: support [-7,1]

- $F_Y(y)=0$ for $y<-7$; $F_Y(y)=(y+7)/8$ for $-7\leq y\leq 1$; $F_Y(y)=1$ for $y>1$.
- $f_Y(y)=1/8$ for $-7\leq y\leq 1$, and 0 otherwise.

4. X is Uniform(-3,3) and $Y=X^2$. Write the support and full piecewise CDF and PDF of Y , accounting for both preimages.

Reference: For $0<y<upper^2$, the event $X^2\leq y$ is $-\sqrt{y}\leq X\leq\sqrt{y}$; both inverse branches contribute to the density.

Answer: support [0,9]

- $F_Y(y)=0$ for $y<0$; $F_Y(y)=\sqrt{y}/3$ for $0\leq y\leq 9$; $F_Y(y)=1$ above 9.
- $f_Y(y)=1/(2(3)\sqrt{y})$ for $0<y<9$, and 0 otherwise. The two $1/(2\sqrt{y})$ Jacobian contributions combine.

Continuous Random Variable Transformations

Worked Answer Key

Write the transformed support, full piecewise CDF, and PDF. Account for decreasing maps and every valid preimage.

5. X is Uniform(0,16) and $Y=\sqrt{X}$. Write the support and full piecewise CDF and PDF of Y.

Reference: The inverse is $x=y^2$. Differentiate $F_X(y^2)$ on the transformed support.

Answer: support [0,4]

1. $F_Y(y)=0$ for $y<0$; $F_Y(y)=y^2/16$ for $0\leq y\leq 4$; $F_Y(y)=1$ for $y>4$.
2. $f_Y(y)=2y/16$ for $0<y<4$, and 0 otherwise.

6. X is Uniform(0,25) and $Y=\sqrt{X}$. Write the support and full piecewise CDF and PDF of Y.

Reference: The inverse is $x=y^2$. Differentiate $F_X(y^2)$ on the transformed support.

Answer: support [0,5]

1. $F_Y(y)=0$ for $y<0$; $F_Y(y)=y^2/25$ for $0\leq y\leq 5$; $F_Y(y)=1$ for $y>5$.
2. $f_Y(y)=2y/25$ for $0<y<5$, and 0 otherwise.

Continuous Random Variable Transformations

Worked Answer Key

Write the transformed support, full piecewise CDF, and PDF. Account for decreasing maps and every valid preimage.

7. X is Uniform(-6,6) and $Y=X^2$. Write the support and full piecewise CDF and PDF of Y, accounting for both preimages.

Reference: For $0 < y < \text{upper}^2$, the event $X^2 \leq y$ is $-\sqrt{y} \leq X \leq \sqrt{y}$; both inverse branches contribute to the density.

Answer: support [0,36]

1. $F_Y(y) = 0$ for $y < 0$; $F_Y(y) = \sqrt{y}/6$ for $0 \leq y \leq 36$; $F_Y(y) = 1$ above 36.
2. $f_Y(y) = 1/(2(6)\sqrt{y})$ for $0 < y < 36$, and 0 otherwise. The two $1/(2\sqrt{y})$ Jacobian contributions combine.

8. X is Uniform(0,7) and $Y=-3X+1$. Write the support and full piecewise CDF and PDF of Y.

Reference: For a monotone map, solve the inequality for X. A negative slope reverses it; the density divides by the absolute derivative.

Answer: support [-20,1]

1. $F_Y(y) = 0$ for $y < -20$; $F_Y(y) = (y+20)/21$ for $-20 \leq y \leq 1$; $F_Y(y) = 1$ for $y > 1$.
2. $f_Y(y) = 1/21$ for $-20 < y < 1$, and 0 otherwise.