

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Continuous PDFs and CDFs

Treat a density value as height, not probability. Verify nonnegativity and total area 1, use definite integrals for interval probabilities and moments, state the full piecewise CDF including outside the support, and remember that a single point has probability 0. Give exact values where practical; otherwise round the final value to four decimals.

1. Is $f(x) = \frac{3}{2000}(x + 1)^2$ on $[-1, 9]$ and 0 otherwise a valid PDF? Justify both validity conditions.

Given: X is measured in units. The displayed formula applies only on the bounded support $[-1, 9]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

2. The CDF is $F(x) = 0$ for $x \leq -1$; $F(x) = \frac{(x + 1)^{11}}{11}$ for $-1 < x < 10$; $F(x) = 1$ for $x \geq 10$. Differentiate it to recover the full PDF.

Given: X is measured in units. The displayed formula applies only on the bounded support $[-1, 10]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

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3. For the PDF $f(x) = 1/8 (x - 2)$ on $[2, 6]$, find $E(X)$.

Given: X is measured in units. The displayed formula applies only on the bounded support $[2, 6]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

4. For $f(x) = 1/10$ on $[-3, 7]$ and 0 otherwise, derive the full piecewise CDF $F(x)$, including both outside-support branches.

Given: X is measured in units. The displayed formula applies only on the bounded support $[-3, 7]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

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5. For $f(x) = \frac{2}{121}(x - 3)$ on $[3, 14]$ and 0 otherwise, find $P(3 \leq X \leq 13)$.

Given: X is measured in units. The displayed formula applies only on the bounded support $[3, 14]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

6. On $-2 \leq x \leq 8$, let $f(x) = k(x + 2)^2$, and let $f(x) = 0$ otherwise. Find k so f is a valid PDF.

Given: X is measured in units. The displayed formula applies only on the bounded support $[-2, 8]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

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7. For the continuous PDF $f(x) = \frac{3}{64}(x + 1)^2$ on $[-1, 3]$, find $P(X = 3)$.

Given: X is measured in units. The displayed formula applies only on the bounded support $[-1, 3]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

8. For the PDF $f(x) = \frac{1}{2}$ on $[0, 2]$, find $\text{Var}(X)$.

Given: X is measured in units. The displayed formula applies only on the bounded support $[0, 2]$; the density is 0 elsewhere. X is continuous, so endpoints and individual points have probability 0.

Continuous PDFs and CDFs

Worked Answer Key

Treat a density value as height, not probability. Verify nonnegativity and total area 1, use definite integrals for interval probabilities and moments, state the full piecewise CDF including outside the support, and remember that a single point has probability 0. Give exact values where practical; otherwise round the final value to four decimals.

1. Is $f(x) = 3/2000 (x + 1)^2$ on $[-1, 9]$ and 0 otherwise a valid PDF? Justify both validity conditions.

Answer: No

1. The formula is nonnegative on $[-1, 9]$.
2. Its total area is 0.5, so the area is not 1.
3. The normalization condition fails.

2. The CDF is $F(x) = 0$ for $x \leq -1$; $F(x) = (x + 1)^{1/11}$ for $-1 < x < 10$; $F(x) = 1$ for $x \geq 10$. Differentiate it to recover the full PDF.

Answer: $f(x) = 1/11$ for $-1 < x < 10$, and $f(x) = 0$ outside the support (endpoint values do not affect probabilities).

1. Differentiate the interior CDF branch: $d/dx [(x + 1)^{1/11}] = 1/11$.
2. The constant outside branches differentiate to 0.
3. A density's values at the two endpoints do not change any integral probability.

Continuous PDFs and CDFs

Worked Answer Key

Treat a density value as height, not probability. Verify nonnegativity and total area 1, use definite integrals for interval probabilities and moments, state the full piecewise CDF including outside the support, and remember that a single point has probability 0. Give exact values where practical; otherwise round the final value to four decimals.

- 3.** For the PDF $f(x) = 1/8 (x - 2)$ on $[2, 6]$, find $E(X)$.

Answer: $14/3$ units (approximately 4.6667)

1. Use $E(X) = \text{integral of } x f(x) \text{ over } [2, 6]$.
2. Equivalently, with $Y = X - 2$, $E(Y) = 4(2)/3$.
3. Thus $E(X) = 2 + E(Y) = 14/3$ units.

- 4.** For $f(x) = 1/10$ on $[-3, 7]$ and 0 otherwise, derive the full piecewise CDF $F(x)$, including both outside-support branches.

Answer: $F(x) = 0$ for $x \leq -3$; $F(x) = (x + 3)^{1/10}$ for $-3 < x < 7$; $F(x) = 1$ for $x \geq 7$.

1. For $x \leq -3$, no density lies to the left, so $F(x) = 0$.
2. For $-3 < x < 7$, integrate $f(t)$ from -3 to x to get $(x + 3)^{1/10}$.
3. For $x \geq 7$, all density has accumulated, so $F(x) = 1$.

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5. For $f(x) = \frac{2}{121}(x - 3)$ on $[3, 14]$ and 0 otherwise, find $P(3 \leq X \leq 13)$.

Answer: $\frac{100}{121}$ (approximately 0.8264)

1. For a continuous variable, integrate the density over the interval.
2. $P = \int_3^{13} \frac{2}{121}(x - 3) dx = \frac{100}{121}$.
3. The final decimal is 0.8264.

6. On $-2 \leq x \leq 8$, let $f(x) = k(x + 2)^2$, and let $f(x) = 0$ otherwise. Find k so f is a valid PDF.

Answer: $k = \frac{3}{1000}$

1. A PDF must be nonnegative and have total area 1.
2. $1 = \int_{-2}^8 k(x + 2)^2 dx = \frac{k(1000)}{3}$.
3. Therefore $k = \frac{3}{1000}$; the polynomial is nonnegative on the support.

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Worked Answer Key

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7. For the continuous PDF $f(x) = \frac{3}{64}(x + 1)^2$ on $[-1, 3]$, find $P(X = 3)$.

Answer: 0

1. A single point has interval width 0.
2. $P(X=3) = \text{integral from 3 to 3 of } f(x) \, dx = 0$.
3. The density height $f(3)$ is not a point probability.

8. For the PDF $f(x) = \frac{1}{2}$ on $[0, 2]$, find $\text{Var}(X)$.

Answer: $\frac{1}{3}$ squared units (approximately 0.3333)

1. Let $Y = X - 0$; shifting does not change variance.
2. $E(Y) = 2(1)/2$ and $E(Y^2) = 4(1)/3$.
3. $\text{Var}(X) = E(Y^2) - [E(Y)]^2 = \frac{1}{3}$ squared units.