

Name _____ Score _____ Teacher _____ Date _____

Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

1. Verify the identity $\sin(5x) + \sin(x) = 2\sin(3x)\cos(2x)$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 9 times its common value.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

2. Verify the identity $\tan(x + \pi/6) = [\tan(x) + \tan(\pi/6)]/[1 - \tan(x)\tan(\pi/6)]$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 5 times its common value.

Assumptions: Work only where the original expression is defined. Every cosine denominator in the original expression must be nonzero. $1 - \tan(x)\tan(\pi/6) \neq 0$.

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Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

3. At $x=15$ degrees, rewrite $\cos(3x)\cos(x)$ with a product-to-sum identity and find its exact value. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. Multiply your final exact value by 8.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

4. Verify the identity $\sin(3x)\cos(x) = [\sin(4x)+\sin(2x)]/2$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 11 times its common value.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

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Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

5. Simplify $[\tan(3x)\cos(3x)][\tan(x)\cos(x)]$ using the named product to sum identity. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. Multiply your final simplified expression by 3.

Assumptions: Work only where the original expression is defined. Every cosine denominator in the original expression must be nonzero.

6. Verify the identity $\sin(x/2) = +\sqrt{[1-\cos(x)]/2}$. Assume $x/2$ lies in Quadrant II. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 1 times its common value.

Assumptions: Work only where the original expression is defined. The sign is determined by the stated quadrant of $x/2$.

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Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

7. At $x=15$ degrees, rewrite $\sin(5x)+\sin(x)$ with a sum-to-product identity and find its exact value. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. Multiply your final exact value by 6.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

8. Verify the identity $\cos(x + \pi/6) = \cos(x)\cos(\pi/6) - \sin(x)\sin(\pi/6)$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 10 times its common value.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

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Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

1. Verify the identity $\sin(5x) + \sin(x) = 2\sin(3x)\cos(2x)$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 9 times its common value.

Answer: $9[2\sin(3x)\cos(2x)]$

1. Begin with the left side $\sin(5x) + \sin(x)$; do not assume the stated equality.
2. Apply the sum to product identity; the left side becomes $2\sin(3x)\cos(2x)$, which is the right side.
3. Use $\sin^2(t) + \cos^2(t) = 1$, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 9; the requested result is $9[2\sin(3x)\cos(2x)]$.

2. Verify the identity $\tan(x + \pi/6) = [\tan(x) + \tan(\pi/6)]/[1 - \tan(x)\tan(\pi/6)]$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 5 times its common value.

Answer: $5[[\tan(x) + \tan(\pi/6)]/[1 - \tan(x)\tan(\pi/6)]]$

1. Begin with the left side $\tan(x + \pi/6)$; do not assume the stated equality.
2. Apply the sum difference identity; the left side becomes $[\tan(x) + \tan(\pi/6)]/[1 - \tan(x)\tan(\pi/6)]$, which is the right side.
3. Use $\sin^2(t) + \cos^2(t) = 1$, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 5; the requested result is $5[[\tan(x) + \tan(\pi/6)]/[1 - \tan(x)\tan(\pi/6)]]$.

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Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

3. At $x=15$ degrees, rewrite $\cos(3x)\cos(x)$ with a product-to-sum identity and find its exact value. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. Multiply your final exact value by 8.

Answer: $8[(\sqrt{3}+1)/4]$

1. Apply the matching product-to-sum identity.
2. Substitute $x=15$ degrees and use exact standard-angle values.
3. Use $\sin^2(t)+\cos^2(t)=1$, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 8; the requested result is $8[(\sqrt{3}+1)/4]$.

4. Verify the identity $\sin(3x)\cos(x) = [\sin(4x)+\sin(2x)]/2$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 11 times its common value.

Answer: $11[(\sin(4x)+\sin(2x))/2]$

1. Begin with the left side $\sin(3x)\cos(x)$; do not assume the stated equality.
2. Apply the product to sum identity; the left side becomes $[\sin(4x)+\sin(2x)]/2$, which is the right side.
3. Use $\sin^2(t)+\cos^2(t)=1$, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 11; the requested result is $11[(\sin(4x)+\sin(2x))/2]$.

Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

5. Simplify $[\tan(3x)\cos(3x)][\tan(x)\cos(x)]$ using the named product to sum identity. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. Multiply your final simplified expression by 3.

Answer: $3[\cos(2x) - \cos(4x)]/2$

1. Start with $[\tan(3x)\cos(3x)][\tan(x)\cos(x)]$.
2. Convert $\tan(u)\cos(u)$ to $\sin(u)$ and retain the original cosine restrictions.
3. Apply the product to sum identity to obtain $[\cos(2x) - \cos(4x)]/2$.
4. Use $\sin^2(t) + \cos^2(t) = 1$, so the added terms cancel and the result is unchanged.
5. Apply the outside factor 3; the requested result is $3[\cos(2x) - \cos(4x)]/2$.

6. Verify the identity $\sin(x/2) = +\sqrt{[1 - \cos(x)]/2}$. Assume $x/2$ lies in Quadrant II. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 1 times its common value.

Answer: $+\sqrt{[1 - \cos(x)]/2}$

1. Begin with the left side $\sin(x/2)$; do not assume the stated equality.
2. Apply the half angle identity; the left side becomes $+\sqrt{[1 - \cos(x)]/2}$, which is the right side.
3. Use $\sin^2(t) + \cos^2(t) = 1$, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 1; the requested result is $+\sqrt{[1 - \cos(x)]/2}$.

Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

7. At $x=15$ degrees, rewrite $\sin(5x)+\sin(x)$ with a sum-to-product identity and find its exact value. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. Multiply your final exact value by 6.

Answer: $6\sqrt{6}/2$

1. Identify the half-sum and half-difference of the arguments.
2. Apply sum-to-product, then substitute $x=15$ degrees.
3. Use $\sin^2(t)+\cos^2(t)=1$, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 6; the requested result is $6\sqrt{6}/2$.

8. Verify the identity $\cos(x + \pi/6) = \cos(x)\cos(\pi/6) - \sin(x)\sin(\pi/6)$. Let E denote the entire requested expression; also simplify $E + \sin^2(t) + \cos^2(t) - 1$. After verifying the equality, report 10 times its common value.

Answer: $10[\cos(x)\cos(\pi/6) - \sin(x)\sin(\pi/6)]$

1. Begin with the left side $\cos(x + \pi/6)$; do not assume the stated equality.
2. Apply the sum difference identity; the left side becomes $\cos(x)\cos(\pi/6) - \sin(x)\sin(\pi/6)$, which is the right side.
3. Use $\sin^2(t)+\cos^2(t)=1$, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 10; the requested result is $10[\cos(x)\cos(\pi/6) - \sin(x)\sin(\pi/6)]$.