Name _____ Score _____ Teacher _____ Date _____

Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

1. Verify the identity sin(5x)+sin(x) = 2sin(3x)cos(2x). Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 9 times its common value.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

2. Verify the identity tan(x + pi/6) = [tan(x)+tan(pi/6)]/[1-tan(x)tan(pi/6)]. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 5 times its common value.

Assumptions: Work only where the original expression is defined. Every cosine denominator in the original expression must be nonzero. 1-tan(x)tan(pi/6) != 0.

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Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

3. At x=15 degrees, rewrite cos(3x)cos(x) with a product-to-sum identity and find its exact value. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. Multiply your final exact value by 8.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

4. Verify the identity sin(3x)cos(x) = [sin(4x)+sin(2x)]/2. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 11 times its common value.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

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Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

5. Simplify [tan(3x)cos(3x)][tan(x)cos(x)] using the named product to sum identity. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. Multiply your final simplified expression by 3.

Assumptions: Work only where the original expression is defined. Every cosine denominator in the original expression must be nonzero.

6. Verify the identity sin(x/2) = +sqrt([1-cos(x)]/2). Assume x/2 lies in Quadrant II. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 1 times its common value.

Assumptions: Work only where the original expression is defined. The sign is determined by the stated quadrant of x/2.

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Advanced Trigonometric Identities

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

7. At x=15 degrees, rewrite sin(5x)+sin(x) with a sum-to-product identity and find its exact value. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. Multiply your final exact value by 6.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

8. Verify the identity cos(x + pi/6) = cos(x)cos(pi/6)-sin(x)sin(pi/6). Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 10 times its common value.

Assumptions: Work only where the original expression is defined. No additional denominator restriction is introduced by this identity.

Name _____ Score _____ Teacher _____ Date _____

Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

1. Verify the identity sin(5x)+sin(x) = 2sin(3x)cos(2x). Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 9 times its common value.

Answer: 9[2sin(3x)cos(2x)]

1. Begin with the left side sin(5x)+sin(x); do not assume the stated equality.
2. Apply the sum to product identity; the left side becomes 2sin(3x)cos(2x), which is the right side.
3. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 9; the requested result is 9[2sin(3x)cos(2x)].

2. Verify the identity tan(x + pi/6) = [tan(x)+tan(pi/6)]/[1-tan(x)tan(pi/6)]. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 5 times its common value.

Answer: 5[[tan(x)+tan(pi/6)]/[1-tan(x)tan(pi/6)]]

1. Begin with the left side tan(x + pi/6); do not assume the stated equality.
2. Apply the sum difference identity; the left side becomes [tan(x)+tan(pi/6)]/[1-tan(x)tan(pi/6)], which is the right side.
3. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 5; the requested result is 5[[tan(x)+tan(pi/6)]/[1-tan(x)tan(pi/6)]].

Name _____ Score _____ Teacher _____ Date _____

Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

3. At x=15 degrees, rewrite cos(3x)cos(x) with a product-to-sum identity and find its exact value. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. Multiply your final exact value by 8.

Answer: 8[(sqrt(3)+1)/4]

1. Apply the matching product-to-sum identity.
2. Substitute x=15 degrees and use exact standard-angle values.
3. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 8; the requested result is 8[(sqrt(3)+1)/4].

4. Verify the identity sin(3x)cos(x) = [sin(4x)+sin(2x)]/2. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 11 times its common value.

Answer: 11[[sin(4x)+sin(2x)]/2]

1. Begin with the left side sin(3x)cos(x); do not assume the stated equality.
2. Apply the product to sum identity; the left side becomes [sin(4x)+sin(2x)]/2, which is the right side.
3. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 11; the requested result is 11[[sin(4x)+sin(2x)]/2].

Name _____ Score _____ Teacher _____ Date _____

Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

5. Simplify [tan(3x)cos(3x)][tan(x)cos(x)] using the named product to sum identity. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. Multiply your final simplified expression by 3.

Answer: 3[[cos(2x)-cos(4x)]/2]

1. Start with [tan(3x)cos(3x)][tan(x)cos(x)].
2. Convert tan(u)cos(u) to sin(u) and retain the original cosine restrictions.
3. Apply the product to sum identity to obtain [cos(2x)-cos(4x)]/2.
4. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
5. Apply the outside factor 3; the requested result is 3[[cos(2x)-cos(4x)]/2].

6. Verify the identity sin(x/2) = +sqrt([1-cos(x)]/2). Assume x/2 lies in Quadrant II. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 1 times its common value.

Answer: +sqrt([1-cos(x)]/2)

1. Begin with the left side sin(x/2); do not assume the stated equality.
2. Apply the half angle identity; the left side becomes +sqrt([1-cos(x)]/2), which is the right side.
3. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 1; the requested result is +sqrt([1-cos(x)]/2).

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Advanced Trigonometric Identities

Worked Answer Key

Name the decisive identity, preserve the original domain, and use the stated quadrant when choosing a half-angle sign.

7. At x=15 degrees, rewrite sin(5x)+sin(x) with a sum-to-product identity and find its exact value. Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. Multiply your final exact value by 6.

Answer: 6[sqrt(6)/2]

1. Identify the half-sum and half-difference of the arguments.
2. Apply sum-to-product, then substitute x=15 degrees.
3. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 6; the requested result is 6[sqrt(6)/2].

8. Verify the identity cos(x + pi/6) = cos(x)cos(pi/6)-sin(x)sin(pi/6). Let E denote the entire requested expression; also simplify E + sin^2(t) + cos^2(t) - 1. After verifying the equality, report 10 times its common value.

Answer: 10[cos(x)cos(pi/6)-sin(x)sin(pi/6)]

1. Begin with the left side cos(x + pi/6); do not assume the stated equality.
2. Apply the sum difference identity; the left side becomes cos(x)cos(pi/6)-sin(x)sin(pi/6), which is the right side.
3. Use sin^2(t)+cos^2(t)=1, so the added terms cancel and the result is unchanged.
4. Apply the outside factor 10; the requested result is 10[cos(x)cos(pi/6)-sin(x)sin(pi/6)].