

Math Sheet

Name _____ Score _____ / 6 Teacher _____ Date _____

Analyzing and Solving Polynomial Equations

For each equation, analyze possible root counts, sign changes and rational candidates, then solve for every exact root with multiplicity.

1. $x^6 + x^5 - 11x^4 + x^3 - 14x^2 - 2x + 24 = 0$

Complex / possible real-nonreal:

$V(P(x))$ / possible positive:

$V(P(-x))$ / possible negative:

Rational candidates:

Exact roots (with multiplicity):

2. $x^4 - 4x^3 + 3x^2 + 4x - 4 = 0$

Complex / possible real-nonreal:

$V(P(x))$ / possible positive:

$V(P(-x))$ / possible negative:

Rational candidates:

Exact roots (with multiplicity):

3. $x^3 + 5x^2 + 16x + 80 = 0$

Complex / possible real-nonreal:

$V(P(x))$ / possible positive:

$V(P(-x))$ / possible negative:

Rational candidates:

Exact roots (with multiplicity):

4. $x^6 - 2x^5 - 26x^4 + 50x^3 + 169x^2 - 288x - 144 = 0$

Complex / possible real-nonreal:

$V(P(x))$ / possible positive:

$V(P(-x))$ / possible negative:

Rational candidates:

Exact roots (with multiplicity):

5. $x^2 - 2x - 15 = 0$

Complex / possible real-nonreal:

$V(P(x))$ / possible positive:

$V(P(-x))$ / possible negative:

Rational candidates:

Exact roots (with multiplicity):

6. $x^5 + 6x^4 + 34x^3 + 96x^2 - 16x - 208 = 0$

Complex / possible real-nonreal:

$V(P(x))$ / possible positive:

$V(P(-x))$ / possible negative:

Rational candidates:

Exact roots (with multiplicity):

Analyzing and Solving Polynomial Equations

Answer key

For each equation, analyze possible root counts, sign changes and rational candidates, then solve for every exact root with multiplicity.

1. $x^6 + x^5 - 11x^4 + x^3 - 14x^2 - 2x + 24 = 0$

Complex: 6; possible (real, nonreal): (6,0), (4,2), (2,4), (0,6)

$V(P(x)) = 4$; possible positive: 4, 2, 0

$V(P(-x)) = 2$; possible negative: 2, 0

Rational candidates: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

Exact roots: $-1, -4, 1, 3, -\sqrt{2}i, \sqrt{2}i$

2. $x^4 - 4x^3 + 3x^2 + 4x - 4 = 0$

Complex: 4; possible (real, nonreal): (4,0), (2,2), (0,4)

$V(P(x)) = 3$; possible positive: 3, 1

$V(P(-x)) = 1$; possible negative: 1

Rational candidates: $\pm 1, \pm 2, \pm 4$

Exact roots: $-1, 1, 2$ (multiplicity 2)

3. $x^3 + 5x^2 + 16x + 80 = 0$

Complex: 3; possible (real, nonreal): (3,0), (1,2)

$V(P(x)) = 0$; possible positive: 0

$V(P(-x)) = 3$; possible negative: 3, 1

Rational candidates: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 16, \pm 20, \pm 40, \pm 80$

Exact roots: $-5, -4i, 4i$

4. $x^6 - 2x^5 - 26x^4 + 50x^3 + 169x^2 - 288x - 144 = 0$

Complex: 6; possible (real, nonreal): (6,0), (4,2), (2,4), (0,6)

$V(P(x)) = 3$; possible positive: 3, 1

$V(P(-x)) = 3$; possible negative: 3, 1

Rational candidates: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 16, \pm 18, \pm 24, \pm 36, \pm 48, \pm 72, \pm 144$

Exact roots: $-3, -4, 3, 4, 1 - \sqrt{2}, 1 + \sqrt{2}$

5. $x^2 - 2x - 15 = 0$

Complex: 2; possible (real, nonreal): (2,0), (0,2)

$V(P(x)) = 1$; possible positive: 1

$V(P(-x)) = 1$; possible negative: 1

Rational candidates: $\pm 1, \pm 3, \pm 5, \pm 15$

Exact roots: $-3, 5$

6. $x^5 + 6x^4 + 34x^3 + 96x^2 - 16x - 208 = 0$

Complex: 5; possible (real, nonreal): (5,0), (3,2), (1,4)

$V(P(x)) = 1$; possible positive: 1

$V(P(-x)) = 4$; possible negative: 4, 2, 0

Rational candidates: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 13, \pm 16, \pm 26, \pm 52, \pm 104, \pm 208$

Exact roots: $-2, -1 - \sqrt{5}, -1 + \sqrt{5}, -1 - 5i, -1 + 5i$