

Math Sheet

Name _____ Score _____ / 12 Teacher _____ Date _____

Irrational and Imaginary Root Theorems

For root counts, use degree and conjugate pairing only; do not solve. Count multiplicity; complex roots include real roots. Then complete root lists and construct monic integer polynomials.

A. Count possible real and nonreal complex roots

1. $x^2 + 4x - 8$

Total complex roots: _____

Possible real/nonreal pairs: _____

2. $x^6 + 2x^5 + 9x^2 + 3x + 8$

Total complex roots: _____

Possible real/nonreal pairs: _____

3. $x^4 - 6x^3 + 7x^2 - 3x - 1$

Total complex roots: _____

Possible real/nonreal pairs: _____

4. $x^3 + 8x^2 + 4x - 1$

Total complex roots: _____

Possible real/nonreal pairs: _____

B. Complete each rational-coefficient root list

5. Given roots: $-1 - \sqrt{3}i$, $1 - \sqrt{3}i$, 4

Missing roots: _____

6. Given roots: $3 - \sqrt{5}i$, $3 + \sqrt{5}i$, $-3 + \sqrt{3}$, $-2 + i$

Missing roots: _____

7. Given roots: $-3 + 2i$, 4 (multiplicity 2), -4

Missing roots: _____

8. Given roots: $-4 + \sqrt{5}$, -4

Missing roots: _____

C. Construct the least-degree monic integer polynomial

9. Given roots: $-3 + \sqrt{3}$

Least-degree monic polynomial: _____

10. Given roots: $2 + 2i$, $2 + \sqrt{2}$

Least-degree monic polynomial: _____

11. Given roots: $3 - 4i$, -4 (multiplicity 2), -2

Least-degree monic polynomial: _____

12. Given roots: $4 - \sqrt{2}i$, -4

Least-degree monic polynomial: _____

Math Sheet

Name _____ Score _____ / 12 Teacher _____ Date _____

Irrational and Imaginary Root Theorems

Answer key

For root counts, use degree and conjugate pairing only; do not solve. Count multiplicity; complex roots include real roots. Then complete root lists and construct monic integer polynomials.

A. Count possible real and nonreal complex roots

1. $x^2 + 4x - 8$
Total complex roots: 2. Possible: (2 real, 0 nonreal), (0 real, 2 nonreal)
2. $x^6 + 2x^5 + 9x^2 + 3x + 8$
Total complex roots: 6. Possible: (6 real, 0 nonreal), (4 real, 2 nonreal), (2 real, 4 nonreal), (0 real, 6 nonreal)
3. $x^4 - 6x^3 + 7x^2 - 3x - 1$
Total complex roots: 4. Possible: (4 real, 0 nonreal), (2 real, 2 nonreal), (0 real, 4 nonreal)
4. $x^3 + 8x^2 + 4x - 1$
Total complex roots: 3. Possible: (3 real, 0 nonreal), (1 real, 2 nonreal)

B. Complete each rational-coefficient root list

5. Given roots: $-1 - \sqrt{3}i$, $1 - \sqrt{3}i$, 4
Missing roots: $-1 + \sqrt{3}i$, $1 + \sqrt{3}i$
6. Given roots: $3 - \sqrt{5}i$, $3 + \sqrt{5}i$, $-3 + \sqrt{3}$, $-2 + i$
Missing roots: $-3 - \sqrt{3}$, $-2 - i$
7. Given roots: $-3 + 2i$, 4 (multiplicity 2), -4
Missing roots: $-3 - 2i$
8. Given roots: $-4 + \sqrt{5}$, -4
Missing roots: $-4 - \sqrt{5}$

C. Construct the least-degree monic integer polynomial

9. Given roots: $-3 + \sqrt{3}$
Least-degree monic polynomial:
 $x^2 + 6x + 6$
10. Given roots: $2 + 2i$, $2 + \sqrt{2}$
Least-degree monic polynomial:
 $x^4 - 8x^3 + 26x^2 - 40x + 16$
11. Given roots: $3 - 4i$, -4 (multiplicity 2), -2
Least-degree monic polynomial:
 $x^5 + 4x^4 - 3x^3 + 90x^2 + 608x + 800$
12. Given roots: $4 - \sqrt{2}i$, -4
Least-degree monic polynomial:
 $x^3 - 4x^2 - 14x + 72$