Original olympiad reasoning practice - Edition 1

Student paper

Name: _____ Date: _____

Write a proof or a complete justified solution, rather than only a numerical answer. Define any new variables you use.

The problems are grouped by increasing demand: four warm-ups, four core arguments, and four stretch proofs. No time limit is prescribed.

Make an independent attempt before using hint 1, and use hint 2 only if still stuck. The teacher edition includes full model arguments.

1. Prove that n^3 - n is divisible by 6 for every integer n.

2. Let x and y be positive real numbers with xy = 1. Prove that x + y >= 2, and determine exactly when equality holds.

Original olympiad reasoning practice - Edition 1

Student paper

Name: _____ Date: _____

Write a proof or a complete justified solution, rather than only a numerical answer. Define any new variables you use.

The problems are grouped by increasing demand: four warm-ups, four core arguments, and four stretch proofs. No time limit is prescribed.

Make an independent attempt before using hint 1, and use hint 2 only if still stuck. The teacher edition includes full model arguments.

3. In a nondegenerate triangle ABC, AB = AC. Let D be the midpoint of BC. Prove that AD is perpendicular to BC and that AD bisects angle BAC.

4. Prove that among any five integers there are two whose difference is divisible by 4. The integers need not be distinct.

Original olympiad reasoning practice - Edition 1

Student paper

Name: _____ Date: _____

Write a proof or a complete justified solution, rather than only a numerical answer. Define any new variables you use.

The problems are grouped by increasing demand: four warm-ups, four core arguments, and four stretch proofs. No time limit is prescribed.

Make an independent attempt before using hint 1, and use hint 2 only if still stuck. The teacher edition includes full model arguments.

5. Find all positive integer pairs (x, y) with x <= y that satisfy 1/x + 1/y = 1/6. Prove that your list is complete.

6. Real numbers a, b, and c satisfy a + b + c = 0. Prove that a^3 + b^3 + c^3 = 3abc. Deduce that if a^3 + b^3 + c^3 = 0 as well, at least one of a, b, c is zero.

Original olympiad reasoning practice - Edition 1

Student paper

Name: _____ Date: _____

Write a proof or a complete justified solution, rather than only a numerical answer. Define any new variables you use.

The problems are grouped by increasing demand: four warm-ups, four core arguments, and four stretch proofs. No time limit is prescribed.

Make an independent attempt before using hint 1, and use hint 2 only if still stuck. The teacher edition includes full model arguments.

7. Triangle ABC is right-angled at A. D is the foot of the perpendicular from A to hypotenuse BC. Write BD = p, DC = q, and AD = h. Prove h^2 = pq.

8. The numbers 1 through 8 are arranged around a circle, each used once. Prove that some two adjacent numbers have sum at least 10. The last and first positions are adjacent.

Original olympiad reasoning practice - Edition 1

Student paper

Name: _____ Date: _____

Write a proof or a complete justified solution, rather than only a numerical answer. Define any new variables you use.

The problems are grouped by increasing demand: four warm-ups, four core arguments, and four stretch proofs. No time limit is prescribed.

Make an independent attempt before using hint 1, and use hint 2 only if still stuck. The teacher edition includes full model arguments.

9. Prove that there are no positive integers a, b, c satisfying a^2 + b^2 = 3c^2.

10. For positive real numbers a, b, and c, prove a/(b + c) + b/(c + a) + c/(a + b) >= 3/2. Determine all equality cases.

Original olympiad reasoning practice - Edition 1

Student paper

Name: _____ Date: _____

Write a proof or a complete justified solution, rather than only a numerical answer. Define any new variables you use.

The problems are grouped by increasing demand: four warm-ups, four core arguments, and four stretch proofs. No time limit is prescribed.

Make an independent attempt before using hint 1, and use hint 2 only if still stuck. The teacher edition includes full model arguments.

11. In an acute triangle ABC, let H be the common intersection of its three altitudes. Let H' be the reflection of H across line BC. Prove that H' lies on the circle through A, B, and C.

12. A 2 by n rectangle is tiled by 1 by 2 dominoes, which may be rotated. Let T_n be the number of tilings, and count the empty 2 by 0 board as having one tiling. Prove that T_n is even exactly when n leaves remainder 2 on division by 3.