

Make an independent attempt before using hint 1, and use hint 2 only if still stuck. The teacher edition includes full model arguments.

1. Prove that $n^3 - n$ is divisible by 6 for every integer n .

2. Let x and y be positive real numbers with $xy = 1$. Prove that $x + y \geq 2$, and determine exactly when equality holds.

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5. Find all positive integer pairs (x, y) with $x \leq y$ that satisfy $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$. Prove that your list is complete.

6. Real numbers a , b , and c satisfy $a + b + c = 0$. Prove that $a^3 + b^3 + c^3 = 3abc$. Deduce that if $a^3 + b^3 + c^3 = 0$ as well, at least one of a , b , c is zero.

7. Triangle ABC is right-angled at A. D is the foot of the perpendicular from A to hypotenuse BC. Write $BD = p$, $DC = q$, and $AD = h$. Prove $h^2 = pq$.

8. The numbers 1 through 8 are arranged around a circle, each used once. Prove that some two adjacent numbers have sum at least 10. The last and first positions are adjacent.

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9. Prove that there are no positive integers a, b, c satisfying $a^2 + b^2 = 3c^2$.

10. For positive real numbers a , b , and c , prove $a/(b + c) + b/(c + a) + c/(a + b) \geq 3/2$. Determine all equality cases.

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11. In an acute triangle ABC , let H be the common intersection of its three altitudes. Let H' be the reflection of H across line BC . Prove that H' lies on the circle through A , B , and C .

12. A 2 by n rectangle is tiled by 1 by 2 dominoes, which may be rotated. Let T_n be the number of tilings, and count the empty 2 by 0 board as having one tiling. Prove that T_n is even exactly when n leaves remainder 2 on division by 3.