

AMC 12-style original practice set - Edition 1

Student paper

Name: _____ Date: _____

Select one answer for each question. Write enough reasoning to check domains, equality cases, and counted cases. No calculator is needed.

This is a fixed 12-question original practice set rather than a full-length AMC 12 exam. No official time limit or score conversion is implied.

Use the separate graduated hints after an attempt; worked solutions are in the teacher edition.

1. For $x > 0$, $\log_2(x) + \log_4(x) = 6$. What is x ?

A. 4 B. 8 C. 16 D. 32 E. 64

2. A real angle θ satisfies $\sin(\theta) + \cos(\theta) = 1/3$. What is $\sin(2\theta)$?

A. $-8/9$ B. $-4/9$ C. $1/9$ D. $4/9$ E. $8/9$

3. Let i be the imaginary unit, so $i^2 = -1$. What is $(1 + i)^8$?

A. -16 B. $-16i$ C. 8 D. 16 E. $16i$

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4. The four real roots of $t^4 - 5t^2 + 4 = 0$ are r_1 , r_2 , r_3 , and r_4 . What is $1/r_1^2 + 1/r_2^2 + 1/r_3^2 + 1/r_4^2$?

A. $5/4$ B. 2 C. $5/2$ D. 4 E. 5

5. How many ordered pairs of integers (a, b) , with $2 \leq a \leq 20$ and $2 \leq b \leq 20$, make $\log_a(b)$ an integer?

A. 19 B. 22 C. 23 D. 24 E. 25

6. Two sides of a triangle have lengths 5 and 7, and their included angle is 120 degrees. What is the circumradius of the triangle?

A. $\sqrt{109}/2$ B. $\sqrt{327}/3$ C. $\sqrt{109}$ D. $35\sqrt{3}/4$ E. $\sqrt{327}$

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7. A nonzero complex number z satisfies $z + 1/z = 1$. What is $z^{2026} + 1/z^{2026}$?

A. -2 B. -1 C. 0 D. 1 E. 2

8. Define $f(x) = (2x + 1)/(x + 2)$, and start with $x_0 = 0$ and $x_{(n+1)} = f(x_n)$. What is x_5 ?

A. 40/41 B. 80/81 C. 120/121 D. 121/122 E. 242/243

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9. Two distinct integers are selected uniformly from 1 through 12, with order ignored. What is the probability that their product is a perfect square?

A. $1/22$ B. $2/33$ C. $5/66$ D. $1/11$ E. $1/6$

10. Real numbers x and y satisfy $x^2/9 + y^2/4 = 1$. What is the greatest possible value of $2x + 3y$?

A. 6 B. $\sqrt{39}$ C. $6\sqrt{2}$ D. 9 E. 12

11. In the expansion of $(x^2 + 1/x)^9$, what is the coefficient of the term independent of x ?

A. 36 B. 72 C. 84 D. 126 E. 252

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12. How many real angles θ in the interval $0 \leq \theta < 2\pi$ satisfy $\sin(3\theta) = \cos(2\theta)$? Angles are measured in radians.

A. 4 B. 5 C. 6 D. 7 E. 10

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Worked solutions

Use after attempting the student paper. Alternative valid methods are welcome.

1. For $x > 0$, $\log_2(x) + \log_4(x) = 6$. What is x ?

A. 4 B. 8 C. 16 D. 32 E. 64

Answer: C. 16

Let $t = \log_2(x)$. Since $4 = 2^2$, $\log_4(x) = t/2$. Thus $3t/2 = 6$, so $t = 4$ and $x = 2^4 = 16$. The positive value 16 satisfies the original equation.

2. A real angle θ satisfies $\sin(\theta) + \cos(\theta) = 1/3$. What is $\sin(2\theta)$?

A. $-8/9$ B. $-4/9$ C. $1/9$ D. $4/9$ E. $8/9$

Answer: A. $-8/9$

Squaring gives $1 + 2 \sin(\theta) \cos(\theta) = 1/9$. Therefore $\sin(2\theta) = 1/9 - 1 = -8/9$. Such angles exist because $\sin(\theta) + \cos(\theta)$ ranges from $-\sqrt{2}$ to $\sqrt{2}$.

3. Let i be the imaginary unit, so $i^2 = -1$. What is $(1 + i)^8$?

A. -16 B. $-16i$ C. 8 D. 16 E. $16i$

Answer: D. 16

We have $(1 + i)^2 = 1 + 2i + i^2 = 2i$. Thus $(1 + i)^4 = (2i)^2 = -4$ and $(1 + i)^8 = (-4)^2 = 16$.

4. The four real roots of $t^4 - 5t^2 + 4 = 0$ are r_1, r_2, r_3 , and r_4 . What is $1/r_1^2 + 1/r_2^2 + 1/r_3^2 + 1/r_4^2$?

A. $5/4$ B. 2 C. $5/2$ D. 4 E. 5

Answer: C. $5/2$

The four roots are $1, -1, 2$, and -2 , all nonzero. The reciprocal squares sum to $1 + 1 + 1/4 + 1/4 = 5/2$.

5. How many ordered pairs of integers (a, b) , with $2 \leq a \leq 20$ and $2 \leq b \leq 20$, make $\log_a(b)$ an integer?

A. 19 B. 22 C. 23 D. 24 E. 25

Answer: D. 24

Since a and b are at least 2, $k = \log_a(b)$ must be a positive integer. For $k = 1$, all 19 bases work. For $k = 2$, the bases are 2, 3, and 4. For $k = 3$ only base 2 works, and for $k = 4$ only base 2 works.

For $k \geq 5$, even 2^k exceeds 20. Each (a, b) has a unique exponent, so the total is $19 + 3 + 1 + 1 = 24$.

6. Two sides of a triangle have lengths 5 and 7, and their included angle is 120 degrees. What is the circumradius of the triangle?

A. $\sqrt{109}/2$ B. $\sqrt{327}/3$ C. $\sqrt{109}$ D. $35\sqrt{3}/4$ E. $\sqrt{327}$

Answer: B. $\sqrt{327}/3$

If c is opposite the 120-degree angle, then $c^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos(120 \text{ degrees}) = 25 + 49 + 35 = 109$.

By the extended sine rule, $R = c/[2 \sin(120 \text{ degrees})] = \sqrt{109}/\sqrt{3} = \sqrt{327}/3$.

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Worked solutions

Use after attempting the student paper. Alternative valid methods are welcome.

7. A nonzero complex number z satisfies $z + 1/z = 1$. What is $z^{2026} + 1/z^{2026}$?

A. -2 B. -1 C. 0 D. 1 E. 2

Answer: B. -1

Multiplying by z gives $z^2 - z + 1 = 0$. Multiplying this by $z + 1$ yields $z^3 + 1 = 0$, so $z^3 = -1$ and $z^6 = 1$. As 2026 has remainder 4 on division by 6, the required sum is $z^4 + z^{(-4)} = z^4 + z^2$.

Using $z^3 = -1$ gives $z^4 = -z$, while $z^2 = z - 1$. Hence the sum is $-z + z - 1 = -1$ for either root.

8. Define $f(x) = (2x + 1)/(x + 2)$, and start with $x_0 = 0$ and $x_{(n+1)} = f(x_n)$. What is x_5 ?

A. 40/41 B. 80/81 C. 120/121 D. 121/122 E. 242/243

Answer: D. 121/122

If $0 \leq x < 1$, then $0 < f(x) < 1$, so all required denominators are nonzero. Direct simplification gives $(1 + f(x))/(1 - f(x)) = 3(1 + x)/(1 - x)$.

Because $T(x_0) = 1$, we have $T(x_5) = 3^5 = 243$. Solving $(1 + x_5)/(1 - x_5) = 243$ gives $244x_5 = 242$, so $x_5 = 121/122$.

9. Two distinct integers are selected uniformly from 1 through 12, with order ignored. What is the probability that their product is a perfect square?

A. 1/22 B. 2/33 C. 5/66 D. 1/11 E. 1/6

Answer: C. 5/66

Write each number as a square times a squarefree integer. The only groups with at least two members are $\{1, 4, 9\}$, with squarefree part 1; $\{2, 8\}$, with part 2; and $\{3, 12\}$, with part 3. All other groups are singletons.

These give $3 + 1 + 1 = 5$ successful pairs. There are $12 \times 11 / 2 = 66$ equally likely distinct unordered pairs, so the probability is $5/66$.

10. Real numbers x and y satisfy $x^2/9 + y^2/4 = 1$. What is the greatest possible value of $2x + 3y$?

A. 6 B. $\sqrt{39}$ C. $6\sqrt{2}$ D. 9 E. 12

Answer: C. $6\sqrt{2}$

Let $u = x/3$ and $v = y/2$. Then $u^2 + v^2 = 1$, and $2x + 3y = 6(u + v)$. Since $(u - v)^2 \geq 0$, $(u + v)^2 \leq 2(u^2 + v^2) = 2$. Therefore $u + v \leq \sqrt{2}$.

Equality is attainable at $u = v = 1/\sqrt{2}$, corresponding to $x = 3/\sqrt{2}$ and $y = 2/\sqrt{2}$. Thus the greatest value is $6\sqrt{2}$.

11. In the expansion of $(x^2 + 1/x)^9$, what is the coefficient of the term independent of x ?

A. 36 B. 72 C. 84 D. 126 E. 252

Answer: C. 84

A term with k choices of x^2 has exponent $2k - (9 - k) = 3k - 9$. This is zero exactly when $k = 3$. Choosing those three factors can be done in $9 \text{ choose } 3 = 9 \times 8 \times 7 / 6 = 84$ ways, each with coefficient 1.

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A. 4 B. 5 C. 6 D. 7 E. 10

Answer: B. 5

The equation becomes $\sin(3\theta) = \sin(\pi/2 - 2\theta)$. Thus either $3\theta = \pi/2 - 2\theta + 2k\pi$, or $3\theta = \pi - (\pi/2 - 2\theta) + 2k\pi$, for an integer k .

The first family is $\theta = \pi/10 + 2k\pi/5$. Exactly $k = 0, 1, 2, 3, 4$ lie in the given interval, giving five angles. The second family is $\theta = \pi/2 + 2k\pi$, whose only angle in the interval is $\pi/2$. It is already the first family with $k = 1$. Thus there are 5 distinct solutions.