

AMC 10-style original practice set - Edition 1

Hint sheet

Try independently first. Read hint 1 before hint 2.

1. What is the value of $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{9 \times 10}$?

A. $\frac{1}{10}$ B. $\frac{4}{5}$ C. $\frac{9}{10}$ D. 1 E. $\frac{9}{5}$

Hint 1: Rewrite each term as a difference of two fractions.

Hint 2: Use $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$.

2. Real numbers x and y satisfy $x + y = 7$ and $xy = 10$. What is $x^2 + y^2$?

A. 19 B. 24 C. 29 D. 39 E. 49

Hint 1: Square $x + y$.

Hint 2: The expansion contains $x^2 + y^2$ plus $2xy$.

3. A chord of a circle has length 12. The perpendicular distance from the center of the circle to the chord is 5. What is the area of the circle?

A. 25 pi B. 36 pi C. 49 pi D. 61 pi E. 169 pi

Hint 1: The perpendicular from the center bisects the chord.

Hint 2: Use a right triangle with legs 5 and 6 and hypotenuse equal to the radius.

4. What is the smallest positive integer that leaves remainder 1 on division by 4, remainder 2 on division by 5, and remainder 3 on division by 6?

A. 17 B. 27 C. 37 D. 57 E. 117

Hint 1: Each remainder is 3 less than its divisor.

Hint 2: Add 3 to the unknown number; the result is divisible by 4, 5, and 6.

5. Three fair six-sided dice are rolled. What is the probability that the largest result is exactly 5?

A. $\frac{1}{6}$ B. $\frac{5}{24}$ C. $\frac{61}{216}$ D. $\frac{25}{72}$ E. $\frac{125}{216}$

Hint 1: Count results whose entries are at most 5.

Hint 2: Subtract those whose entries are all at most 4.

6. The roots of $t^2 - 5t + 3 = 0$ are r and s . What is $r^3 + s^3$?

A. 35 B. 50 C. 65 D. 80 E. 125

Hint 1: The sum and product of the roots are 5 and 3.

Hint 2: Expand $(r + s)^3$ and group the mixed terms as $3rs(r + s)$.

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7. Triangle ABC has area 84. D is the midpoint of BC. E lies on AC with $AE:EC = 1:2$. Segments AD and BE meet at P. What is the area of triangle ABP?

A. 14 B. 21 C. 28 D. 35 E. 42

Hint 1: Use coordinates based on the two sides from A; area ratios do not depend on the triangle's shape.

Hint 2: Set $A = (0, 0)$, $B = (1, 0)$, $C = (0, 1)$. Then $D = (1/2, 1/2)$ and $E = (0, 1/3)$.

8. How many length-six strings made from the letters A, B, and C contain an even number of each letter? Zero occurrences count as even.

A. 90 B. 123 C. 180 D. 183 E. 243

Hint 1: The possible occurrence-count patterns are $(6,0,0)$, $(4,2,0)$, and $(2,2,2)$, in any order.

Hint 2: Count the choices of letters as well as their positions.

9. How many three-digit numbers use three different digits from 1, 2, 3, 4, 5, and 6 and are divisible by 3?

A. 24 B. 36 C. 48 D. 60 E. 72

Hint 1: Group the digits by their remainder on division by 3.

Hint 2: Each remainder class has only two digits, so a valid triple must use one from each class.

10. Real numbers x , y , and z satisfy $x + y + z = 0$ and $x^2 + y^2 + z^2 = 14$. What is $x^4 + y^4 + z^4$?

A. 49 B. 70 C. 84 D. 98 E. 196

Hint 1: First find $xy + yz + zx$ by squaring $x + y + z$.

Hint 2: Square $xy + yz + zx$; the extra mixed term contains $xyz(x + y + z)$.

11. A sequence starts with $a_1 = 2$ and satisfies $a_{(n+1)} = 1/(1 - a_n)$. What is a_{2026} ?

A. -1 B. 0 C. $1/2$ D. 1 E. 2

Hint 1: Calculate the next three terms.

Hint 2: Check that the terms 2, -1, and $1/2$ repeat, and none makes a denominator zero.

12. Four red counters and four blue counters are placed in a uniformly random order. What is the probability that every initial segment of the order contains at least as many red counters as blue counters?

A. $1/14$ B. $1/7$ C. $1/5$ D. $2/7$ E. $1/2$

Hint 1: There are 70 equally likely color orders. Count the orders that first let blue move ahead.

Hint 2: For a bad order, interchange red and blue up to and including the first prefix with one more blue than red.