

## AMC 10-style original practice set - Edition 1

Student paper

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Choose one answer per question and justify it on the paper. Work without a calculator.

This fixed 12-question edition is a practice selection, not a full-length AMC 10 exam. There is no official timing or score conversion.

Try each problem before opening the two-stage hints. Complete worked solutions are separate from the student pages.

**1. What is the value of  $1/(1 \times 2) + 1/(2 \times 3) + 1/(3 \times 4) + \dots + 1/(9 \times 10)$ ?**

A.  $1/10$  B.  $4/5$  C.  $9/10$  D. 1 E.  $9/5$

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**2. Real numbers  $x$  and  $y$  satisfy  $x + y = 7$  and  $xy = 10$ . What is  $x^2 + y^2$ ?**

A. 19 B. 24 C. 29 D. 39 E. 49

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**3. A chord of a circle has length 12. The perpendicular distance from the center of the circle to the chord is 5. What is the area of the circle?**

A. 25 pi B. 36 pi C. 49 pi D. 61 pi E. 169 pi

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**4. What is the smallest positive integer that leaves remainder 1 on division by 4, remainder 2 on division by 5, and remainder 3 on division by 6?**

A. 17 B. 27 C. 37 D. 57 E. 117

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**5. Three fair six-sided dice are rolled. What is the probability that the largest result is exactly 5?**

A.  $\frac{1}{6}$  B.  $\frac{5}{24}$  C.  $\frac{61}{216}$  D.  $\frac{25}{72}$  E.  $\frac{125}{216}$

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**6. The roots of  $t^2 - 5t + 3 = 0$  are  $r$  and  $s$ . What is  $r^3 + s^3$ ?**

A. 35 B. 50 C. 65 D. 80 E. 125

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**7. Triangle ABC has area 84. D is the midpoint of BC. E lies on AC with  $AE:EC = 1:2$ . Segments AD and BE meet at P. What is the area of triangle ABP?**

A. 14 B. 21 C. 28 D. 35 E. 42

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**8. How many length-six strings made from the letters A, B, and C contain an even number of each letter? Zero occurrences count as even.**

A. 90 B. 123 C. 180 D. 183 E. 243

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**9. How many three-digit numbers use three different digits from 1, 2, 3, 4, 5, and 6 and are divisible by 3?**

A. 24 B. 36 C. 48 D. 60 E. 72

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**10. Real numbers  $x$ ,  $y$ , and  $z$  satisfy  $x + y + z = 0$  and  $x^2 + y^2 + z^2 = 14$ . What is  $x^4 + y^4 + z^4$ ?**

A. 49 B. 70 C. 84 D. 98 E. 196

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**11. A sequence starts with  $a_1 = 2$  and satisfies  $a_{n+1} = 1/(1 - a_n)$ . What is  $a_{2026}$ ?**

A. -1 B. 0 C.  $1/2$  D. 1 E. 2

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Try each problem before opening the two-stage hints. Complete worked solutions are separate from the student pages.

**12. Four red counters and four blue counters are placed in a uniformly random order. What is the probability that every initial segment of the order contains at least as many red counters as blue counters?**

A.  $\frac{1}{14}$  B.  $\frac{1}{7}$  C.  $\frac{1}{5}$  D.  $\frac{2}{7}$  E.  $\frac{1}{2}$

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## AMC 10-style original practice set - Edition 1

Worked solutions

Use after attempting the student paper. Alternative valid methods are welcome.

**1. What is the value of  $1/(1 \times 2) + 1/(2 \times 3) + 1/(3 \times 4) + \dots + 1/(9 \times 10)$ ?**

A.  $1/10$  B.  $4/5$  C.  $9/10$  D. 1 E.  $9/5$

**Answer: C.  $9/10$**

The sum is  $(1 - 1/2) + (1/2 - 1/3) + \dots + (1/9 - 1/10)$ . All intermediate fractions cancel, leaving  $1 - 1/10 = 9/10$ .

**2. Real numbers  $x$  and  $y$  satisfy  $x + y = 7$  and  $xy = 10$ . What is  $x^2 + y^2$ ?**

A. 19 B. 24 C. 29 D. 39 E. 49

**Answer: C. 29**

The identity  $(x + y)^2 = x^2 + y^2 + 2xy$  gives  $x^2 + y^2 = 49 - 20 = 29$ . The given conditions are consistent, for example  $x = 2$  and  $y = 5$ .

**3. A chord of a circle has length 12. The perpendicular distance from the center of the circle to the chord is 5. What is the area of the circle?**

A. 25 pi B. 36 pi C. 49 pi D. 61 pi E. 169 pi

**Answer: D. 61 pi**

The half-chord has length 6. By the Pythagorean theorem, the radius satisfies  $r^2 = 5^2 + 6^2 = 61$ . Thus the circle has area  $\pi r^2 = 61 \pi$ .

**4. What is the smallest positive integer that leaves remainder 1 on division by 4, remainder 2 on division by 5, and remainder 3 on division by 6?**

A. 17 B. 27 C. 37 D. 57 E. 117

**Answer: D. 57**

If  $n$  is the required number,  $n + 3$  is a positive common multiple of 4, 5, and 6. Their least common multiple is 60, so the smallest positive candidate is  $n = 60 - 3 = 57$ . It gives the three required remainders.

**5. Three fair six-sided dice are rolled. What is the probability that the largest result is exactly 5?**

A.  $1/6$  B.  $5/24$  C.  $61/216$  D.  $25/72$  E.  $125/216$

**Answer: C.  $61/216$**

There are  $6^3 = 216$  equally likely ordered results. Exactly  $5^3 - 4^3 = 125 - 64 = 61$  have all entries at most 5 and at least one entry equal to 5. Therefore the probability is  $61/216$ .

**6. The roots of  $t^2 - 5t + 3 = 0$  are  $r$  and  $s$ . What is  $r^3 + s^3$ ?**

A. 35 B. 50 C. 65 D. 80 E. 125

**Answer: D. 80**

From the coefficients,  $r + s = 5$  and  $rs = 3$ . Thus  $r^3 + s^3 = (r + s)^3 - 3rs(r + s) = 125 - 45 = 80$ .

# AMC 10-style original practice set - Edition 1

Worked solutions

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**7. Triangle ABC has area 84. D is the midpoint of BC. E lies on AC with  $AE:EC = 1:2$ . Segments AD and BE meet at P. What is the area of triangle ABP?**

A. 14 B. 21 C. 28 D. 35 E. 42

**Answer: B. 21**

Use oblique coordinates with vectors AB and AC as basis vectors. Then  $D = (1/2, 1/2)$ ,  $E = (0, 1/3)$ , and points on AD have equal coordinates. A point on BE has coordinates  $(1 - t, t/3)$ . Equating them gives  $1 - t = t/3$ , so  $t = 3/4$  and  $P = (1/4, 1/4)$ .

Triangles ABP and ABC share base AB. The altitude of P above AB is  $1/4$  the altitude of C, because the AC-coordinate is  $1/4$ . Their area ratio is  $1/4$ , so the required area is  $84/4 = 21$ .

**8. How many length-six strings made from the letters A, B, and C contain an even number of each letter? Zero occurrences count as even.**

A. 90 B. 123 C. 180 D. 183 E. 243

**Answer: D. 183**

For pattern (6,0,0), choose the repeated letter in 3 ways. For pattern (4,2,0), choose the letter used four times in 3 ways and the letter used twice in 2 ways; choose its two positions in 15 ways. This gives  $3 \times 2 \times 15 = 90$ .

For pattern (2,2,2), choose the two A positions in 15 ways and the two B positions among the remaining four in 6 ways; C fills the rest. This gives 90. The disjoint cases total  $3 + 90 + 90 = 183$ .

**9. How many three-digit numbers use three different digits from 1, 2, 3, 4, 5, and 6 and are divisible by 3?**

A. 24 B. 36 C. 48 D. 60 E. 72

**Answer: C. 48**

The remainder classes are  $\{3,6\}$ ,  $\{1,4\}$ , and  $\{2,5\}$ . A sum of three remainders is 0 modulo 3 only if all three remainders are equal or all are different. Equal remainders would require three digits from a class of size two, which is impossible.

Choose one digit from each class in  $2 \times 2 \times 2 = 8$  ways. Each triple has 6 orders, all producing three-digit numbers, so the answer is  $8 \times 6 = 48$ .

**10. Real numbers x, y, and z satisfy  $x + y + z = 0$  and  $x^2 + y^2 + z^2 = 14$ . What is  $x^4 + y^4 + z^4$ ?**

A. 49 B. 70 C. 84 D. 98 E. 196

**Answer: D. 98**

Squaring  $x + y + z = 0$  gives  $xy + yz + zx = -7$ . Squaring this relation gives  $x^2 y^2 + y^2 z^2 + z^2 x^2 + 2xyz(x + y + z) = 49$ . The last term is zero, so the sum of pairwise squared products is 49.

Now  $14^2 = (x^2 + y^2 + z^2)^2 = x^4 + y^4 + z^4 + 2 \times 49$ . Hence  $x^4 + y^4 + z^4 = 196 - 98 = 98$ .

**11. A sequence starts with  $a_1 = 2$  and satisfies  $a_{n+1} = 1/(1 - a_n)$ . What is  $a_{2026}$ ?**

A. -1 B. 0 C.  $1/2$  D. 1 E. 2

**Answer: E. 2**

Starting at 2 gives  $a_2 = -1$ ,  $a_3 = 1/2$ , and  $a_4 = 2$ . Since the recurrence is deterministic and all three denominators are nonzero, these three values repeat forever. Because  $2026 = 3 \times 675 + 1$ ,  $a_{2026} = a_1 = 2$ .

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A.  $1/14$  B.  $1/7$  C.  $1/5$  D.  $2/7$  E.  $1/2$

**Answer: C.  $1/5$**

There are  $\binom{8}{4} = 70$  color orders. For each bad order, find the first prefix with one more blue than red, and swap both colors throughout that prefix. The resulting order has 5 red and 3 blue counters.

This is a bijection: every order with 5 red and 3 blue first reaches a prefix with one more red than blue; swapping that prefix recovers a unique bad order with 4 of each color. There are  $\binom{8}{3} = 56$  such orders, so  $70 - 56 = 14$  are good. The probability is  $14/70 = 1/5$ .