

# Math Sheet

Name \_\_\_\_\_ Score \_\_\_\_\_ / 12 Teacher \_\_\_\_\_ Date \_\_\_\_\_

## Cramer's Rule - 3 x 3

Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

1. Solve the system.

$$-5x + 7y - 3z = -40$$

$$-3x - y + z = -4$$

$$-x - 6y - 4z = 50$$

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2. Solve the system.

$$-2x - 4y - 2z = 0$$

$$-2x - 4y - 2z = 0$$

$$-x - 2y - z = 1$$

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3. Solve the system.

$$x - 3y - z = 14$$

$$5x + 7y - 9z = 0$$

$$5x - 3y + z = 40$$

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4. Solve the system.

$$4x - y + z = -3$$

$$x - y + 4z = -12$$

$$5x - 2y + 5z = -15$$

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5. Solve the system.

$$-5x + 7y - 7z = 44$$

$$-2x + 9y + 3z = 1$$

$$-x - y - 6z = 27$$

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6. Solve the system.

$$9x - 3y + 6z = 39$$

$$-9x + 3y - 6z = -39$$

$$-9x + 3y - 6z = -39$$

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Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

7. Solve the system.

$$-9x - 2y + 6z = -12$$

$$-5x - 4y - 7z = -60$$

$$x + 6y - 6z = -48$$

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8. Solve the system.

$$2x + 2y + 7z = -40$$

$$8x - 9y - 8z = -16$$

$$8x - 3y - 9z = -12$$

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9. Solve the system.

$$-3x + 4y - 3z = -7$$

$$-4x + 4y - 3z = -10$$

$$-7x + 8y - 6z = -16$$

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10. Solve the system.

$$6x - 9y + 3z = -54$$

$$-2x + 3y - z = 18$$

$$-2x + 3y - z = 19$$

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11. Solve the system.

$$x - 4y - 2z = -7$$

$$-5x - 7y + 8z = 8$$

$$-5x - 2y + 7z = 13$$

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12. Solve the system.

$$2x - y - 4z = -22$$

$$-x + 3y - 2z = 1$$

$$x + 2y - 6z = -21$$

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## Cramer's Rule - 3 x 3

### Answer Key

Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

1. Solve the system.

$$-5x + 7y - 3z = -40$$

$$-3x - y + z = -4$$

$$-x - 6y - 4z = 50$$

Replace one coefficient column at a time with the constants.

$$D = \begin{vmatrix} -5 & 7 & -3 \\ -3 & -1 & 1 \\ -1 & -6 & -4 \end{vmatrix} = -192 \quad D_x = \begin{vmatrix} -40 & 7 & -3 \\ -4 & -1 & 1 \\ 50 & -6 & -4 \end{vmatrix} = -384 \quad D_y = \begin{vmatrix} -5 & -40 & -3 \\ -3 & -4 & 1 \\ -1 & 50 & -4 \end{vmatrix} = 1152$$
$$D_z = \begin{vmatrix} -5 & 7 & -40 \\ -3 & -1 & -4 \\ -1 & -6 & 50 \end{vmatrix} = 768$$

D is nonzero. Divide each replacement determinant by D.

$$x = \frac{-384}{-192} = 2 \quad y = \frac{1152}{-192} = -6 \quad z = \frac{768}{-192} = -4$$

2. Solve the system.

$$-2x - 4y - 2z = 0$$

$$-2x - 4y - 2z = 0$$

$$-x - 2y - z = 1$$

$$D = \det(A) = 0$$

D is zero. Row-reduce the augmented matrix to check consistency.

$$\left[ \begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{rank}(A) = 1; \text{rank}([A|b]) = 2.$$

$$0 = 1$$

No solution: the reduced system contains a contradiction.

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## Cramer's Rule - 3 x 3

### Answer Key

Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

3. Solve the system.

$$x - 3y - z = 14$$

$$5x + 7y - 9z = 0$$

$$5x - 3y + z = 40$$

Replace one coefficient column at a time with the constants.

$$D = \begin{vmatrix} 1 & -3 & -1 \\ 5 & 7 & -9 \\ 5 & -3 & 1 \end{vmatrix} = 180 \quad D_x = \begin{vmatrix} 14 & -3 & -1 \\ 0 & 7 & -9 \\ 40 & -3 & 1 \end{vmatrix} = 1080 \quad D_y = \begin{vmatrix} 1 & 14 & -1 \\ 5 & 0 & -9 \\ 5 & 40 & 1 \end{vmatrix} = -540$$
$$D_z = \begin{vmatrix} 1 & -3 & 14 \\ 5 & 7 & 0 \\ 5 & -3 & 40 \end{vmatrix} = 180$$

D is nonzero. Divide each replacement determinant by D.

$$x = \frac{1080}{180} = 6 \quad y = \frac{-540}{180} = -3 \quad z = \frac{180}{180} = 1$$

4. Solve the system.

$$4x - y + z = -3$$

$$x - y + 4z = -12$$

$$5x - 2y + 5z = -15$$

$$D = \det(A) = 0$$

D is zero. Row-reduce the augmented matrix to check consistency.

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & -5 & 15 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{rank}(A) = 2; \text{rank}([A|b]) = 2.$$

Free variables: z

Infinitely many solutions: the system is consistent and has a free variable.

# Math Sheet

Name \_\_\_\_\_ Score \_\_\_\_\_ / 12 Teacher \_\_\_\_\_ Date \_\_\_\_\_

## Cramer's Rule - 3 x 3

### Answer Key

Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

5. Solve the system.

$$-5x + 7y - 7z = 44$$

$$-2x + 9y + 3z = 1$$

$$-x - y - 6z = 27$$

Replace one coefficient column at a time with the constants.

$$D = \begin{vmatrix} -5 & 7 & -7 \\ -2 & 9 & 3 \\ -1 & -1 & -6 \end{vmatrix} = 73 \quad D_x = \begin{vmatrix} 44 & 7 & -7 \\ 1 & 9 & 3 \\ 27 & -1 & -6 \end{vmatrix} = 73 \quad D_y = \begin{vmatrix} -5 & 44 & -7 \\ -2 & 1 & 3 \\ -1 & 27 & -6 \end{vmatrix} = 146$$
$$D_z = \begin{vmatrix} -5 & 7 & 44 \\ -2 & 9 & 1 \\ -1 & -1 & 27 \end{vmatrix} = -365$$

D is nonzero. Divide each replacement determinant by D.

$$x = \frac{73}{73} = 1 \quad y = \frac{146}{73} = 2 \quad z = \frac{-365}{73} = -5$$

6. Solve the system.

$$9x - 3y + 6z = 39$$

$$-9x + 3y - 6z = -39$$

$$-9x + 3y - 6z = -39$$

$$D = \det(A) = 0$$

D is zero. Row-reduce the augmented matrix to check consistency.

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{2}{3} & \frac{13}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{rank}(A) = 1; \text{rank}([A|b]) = 1.$$

Free variables:  $y, z$

Infinitely many solutions: the system is consistent and has a free variable.

## Cramer's Rule - 3 x 3

### Answer Key

Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

7. Solve the system.

$$-9x - 2y + 6z = -12$$

$$-5x - 4y - 7z = -60$$

$$x + 6y - 6z = -48$$

Replace one coefficient column at a time with the constants.

$$D = \begin{vmatrix} -9 & -2 & 6 \\ -5 & -4 & -7 \\ 1 & 6 & -6 \end{vmatrix} = -676 \quad D_x = \begin{vmatrix} -12 & -2 & 6 \\ -60 & -4 & -7 \\ -48 & 6 & -6 \end{vmatrix} = -4056 \quad D_y = \begin{vmatrix} -9 & -12 & 6 \\ -5 & -60 & -7 \\ 1 & -48 & -6 \end{vmatrix} = 2028$$

$$D_z = \begin{vmatrix} -9 & -2 & -12 \\ -5 & -4 & -60 \\ 1 & 6 & -48 \end{vmatrix} = -4056$$

D is nonzero. Divide each replacement determinant by D.

$$x = \frac{-4056}{-676} = 6 \quad y = \frac{2028}{-676} = -3 \quad z = \frac{-4056}{-676} = 6$$

## Cramer's Rule - 3 x 3

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Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

8. Solve the system.

$$2x + 2y + 7z = -40$$

$$8x - 9y - 8z = -16$$

$$8x - 3y - 9z = -12$$

Replace one coefficient column at a time with the constants.

$$D = \begin{vmatrix} 2 & 2 & 7 \\ 8 & -9 & -8 \\ 8 & -3 & -9 \end{vmatrix} = 466 \quad D_x = \begin{vmatrix} -40 & 2 & 7 \\ -16 & -9 & -8 \\ -12 & -3 & -9 \end{vmatrix} = -2796 \quad D_y = \begin{vmatrix} 2 & -40 & 7 \\ 8 & -16 & -8 \\ 8 & -12 & -9 \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} 2 & 2 & -40 \\ 8 & -9 & -16 \\ 8 & -3 & -12 \end{vmatrix} = -1864$$

$D$  is nonzero. Divide each replacement determinant by  $D$ .

$$x = \frac{-2796}{466} = -6 \quad y = \frac{0}{466} = 0 \quad z = \frac{-1864}{466} = -4$$

9. Solve the system.

$$-3x + 4y - 3z = -7$$

$$-4x + 4y - 3z = -10$$

$$-7x + 8y - 6z = -16$$

$$D = \det(A) = 0$$

$D$  is zero. Row-reduce the augmented matrix to check consistency.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$\text{rank}(A) = 2; \text{rank}([A|b]) = 3.$$

$$0 = 1$$

No solution: the reduced system contains a contradiction.

## Cramer's Rule - 3 x 3

### Answer Key

Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

10. Solve the system.

$$6x - 9y + 3z = -54$$

$$-2x + 3y - z = 18$$

$$-2x + 3y - z = 19$$

$$D = \det(A) = 0$$

$D$  is zero. Row-reduce the augmented matrix to check consistency.

$$\left[ \begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{rank}(A) = 1; \text{rank}([A|b]) = 2.$$

$$0 = 1$$

No solution: the reduced system contains a contradiction.

11. Solve the system.

$$x - 4y - 2z = -7$$

$$-5x - 7y + 8z = 8$$

$$-5x - 2y + 7z = 13$$

Replace one coefficient column at a time with the constants.

$$D = \begin{vmatrix} 1 & -4 & -2 \\ -5 & -7 & 8 \\ -5 & -2 & 7 \end{vmatrix} = 37 \quad D_x = \begin{vmatrix} -7 & -4 & -2 \\ 8 & -7 & 8 \\ 13 & -2 & 7 \end{vmatrix} = -111 \quad D_y = \begin{vmatrix} 1 & -7 & -2 \\ -5 & 8 & 8 \\ -5 & 13 & 7 \end{vmatrix} = 37$$

$$D_z = \begin{vmatrix} 1 & -4 & -7 \\ -5 & -7 & 8 \\ -5 & -2 & 13 \end{vmatrix} = 0$$

$D$  is nonzero. Divide each replacement determinant by  $D$ .

$$x = \frac{-111}{37} = -3 \quad y = \frac{37}{37} = 1 \quad z = \frac{0}{37} = 0$$

## Cramer's Rule - 3 x 3

### Answer Key

Solve each system. If  $D = 0$ , determine whether there is no solution or infinitely many solutions.

12. Solve the system.

$$2x - y - 4z = -22$$

$$-x + 3y - 2z = 1$$

$$x + 2y - 6z = -21$$

$$D = \det(A) = 0$$

D is zero. Row-reduce the augmented matrix to check consistency.

$$\left[ \begin{array}{ccc|c} 1 & 0 & -\frac{14}{5} & -13 \\ 0 & 1 & -\frac{8}{5} & -4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{rank}(A) = 2; \text{rank}([A|b]) = 2.$$

Free variables:  $z$

Infinitely many solutions: the system is consistent and has a free variable.