

# Math Sheet

Name \_\_\_\_\_ Score \_\_\_\_\_ Teacher \_\_\_\_\_ Date \_\_\_\_\_

## Work by Integration

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

1. A spring has constant 44 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.

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2. A full rectangular tank has horizontal area  $5 \text{ m}^2$  and depth 3 m. Pump water to the top (weight density  $9800 \text{ N/m}^3$ ). Find the work.

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Work by Integration

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

3. A 5 m cable weighs 2 N/m and hangs vertically. Set up a definite integral for the work to lift all of it to the top.

4. A 7 m cable weighs 3 N/m and hangs vertically. Find the work to lift all of it to the top.

# Math Sheet

Name \_\_\_\_\_ Score \_\_\_\_\_ Teacher \_\_\_\_\_ Date \_\_\_\_\_

## Work by Integration

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

5. A spring has constant 53 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.

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6. A full rectangular tank has horizontal area  $3 \text{ m}^2$  and depth 2 m. Pump water to the top (weight density  $9800 \text{ N/m}^3$ ). Find the work.

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# Math Sheet

Name \_\_\_\_\_ Score \_\_\_\_\_ Teacher \_\_\_\_\_ Date \_\_\_\_\_

## Work by Integration

### Worked Answer Key

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

1. A spring has constant 44 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.

**Answer: integral from 0.2 to 0.5  $44x \, dx$  J.**

1. Hooke's law gives  $F(x)=44x$  N.
2.  $W=\text{integral from 0.2 to 0.5 } 44x \, dx$ .
3. The force-distance integral is the requested setup.

2. A full rectangular tank has horizontal area  $5 \, \text{m}^2$  and depth 3 m. Pump water to the top (weight density  $9800 \, \text{N/m}^3$ ). Find the work.

**Answer: 220500 J.**

1. A horizontal slice at depth  $y$  has volume  $5dy$  and moves  $y$  meters.
2.  $dW=9800(5)y \, dy$ , so  $W=9800(5) \text{ integral from 0 to 3 } y \, dy$ .
3.  $W=9800(5)(3^2)/2=220500$  J.

## Work by Integration

### Worked Answer Key

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

- 3. A 5 m cable weighs 2 N/m and hangs vertically. Set up a definite integral for the work to lift all of it to the top.**

**Answer: integral from 0 to 5  $2x \, dx$  J.**

1. A slice  $x$  meters below the top moves  $x$  meters and weighs  $2 \, dx$  N.
2.  $dW = 2x \, dx$ , so  $W = \text{integral from 0 to 5 } 2x \, dx$ .
3. The force-distance integral is the requested setup.

- 4. A 7 m cable weighs 3 N/m and hangs vertically. Find the work to lift all of it to the top.**

**Answer: 73.5 J.**

1. A slice  $x$  meters below the top moves  $x$  meters and weighs  $3 \, dx$  N.
2.  $dW = 3x \, dx$ , so  $W = \text{integral from 0 to 7 } 3x \, dx$ .
3.  $W = 3(7^2)/2 = 73.5 \, \text{J}$ .

# Math Sheet

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## Work by Integration

### Worked Answer Key

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

5. A spring has constant 53 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.

**Answer: integral from 0.2 to 0.5  $53x \, dx$  J.**

1. Hooke's law gives  $F(x)=53x$  N.
2.  $W=\text{integral from 0.2 to 0.5 } 53x \, dx$ .
3. The force-distance integral is the requested setup.

6. A full rectangular tank has horizontal area  $3 \, \text{m}^2$  and depth 2 m. Pump water to the top (weight density  $9800 \, \text{N/m}^3$ ). Find the work.

**Answer: 58800 J.**

1. A horizontal slice at depth  $y$  has volume  $3dy$  and moves  $y$  meters.
2.  $dW=9800(3)y \, dy$ , so  $W=9800(3) \text{ integral from 0 to 2 } y \, dy$ .
3.  $W=9800(3)(2^2)/2=58800$  J.