

Work by Integration

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

1. A spring has constant 44 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.

2. A full rectangular tank has horizontal area 5 m^2 and depth 3 m. Pump water to the top (weight density 9800 N/m^3). Find the work.

Work by Integration

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

3. A 5 m cable weighs 2 N/m and hangs vertically. Set up a definite integral for the work to lift all of it to the top.

4. A 7 m cable weighs 3 N/m and hangs vertically. Find the work to lift all of it to the top.

Work by Integration

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

5. A spring has constant 53 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.

6. A full rectangular tank has horizontal area 3 m² and depth 2 m. Pump water to the top (weight density 9800 N/m³). Find the work.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Work by Integration

Worked Answer Key

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

- 1. A spring has constant 44 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.**

Answer: integral from 0.2 to 0.5 $44x \, dx$ J.

1. Hooke's law gives $F(x)=44x$ N.
2. $W=\text{integral from 0.2 to 0.5 } 44x \, dx$.
3. The force-distance integral is the requested setup.

- 2. A full rectangular tank has horizontal area $5 \, \text{m}^2$ and depth 3 m. Pump water to the top (weight density $9800 \, \text{N/m}^3$). Find the work.**

Answer: 220500 J.

1. A horizontal slice at depth y has volume $5dy$ and moves y meters.
2. $dW=9800(5)y \, dy$, so $W=9800(5) \text{ integral from 0 to 3 } y \, dy$.
3. $W=9800(5)(3^2)/2=220500$ J.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Work by Integration

Worked Answer Key

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

- 3. A 5 m cable weighs 2 N/m and hangs vertically. Set up a definite integral for the work to lift all of it to the top.**

Answer: integral from 0 to 5 $2x \, dx$ J.

1. A slice x meters below the top moves x meters and weighs $2 \, dx$ N.
2. $dW = 2x \, dx$, so $W = \text{integral from 0 to 5 } 2x \, dx$.
3. The force-distance integral is the requested setup.

- 4. A 7 m cable weighs 3 N/m and hangs vertically. Find the work to lift all of it to the top.**

Answer: 73.5 J.

1. A slice x meters below the top moves x meters and weighs $3 \, dx$ N.
2. $dW = 3x \, dx$, so $W = \text{integral from 0 to 7 } 3x \, dx$.
3. $W = 3(7^2)/2 = 73.5$ J.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Work by Integration

Worked Answer Key

Define the force at a general displacement or slice, multiply by distance moved, and evaluate with physical units.

5. A spring has constant 53 N/m. Set up a definite integral for the work to stretch it from 0.2 m to 0.5 m.

Answer: integral from 0.2 to 0.5 $53x \, dx$ J.

1. Hooke's law gives $F(x)=53x$ N.
2. $W=\text{integral from 0.2 to 0.5 } 53x \, dx$.
3. The force-distance integral is the requested setup.

6. A full rectangular tank has horizontal area $3 \, \text{m}^2$ and depth 2 m. Pump water to the top (weight density $9800 \, \text{N/m}^3$). Find the work.

Answer: 58800 J.

1. A horizontal slice at depth y has volume $3dy$ and moves y meters.
2. $dW=9800(3)y \, dy$, so $W=9800(3) \text{ integral from 0 to 2 } y \, dy$.
3. $W=9800(3)(2^2)/2=58800$ J.