

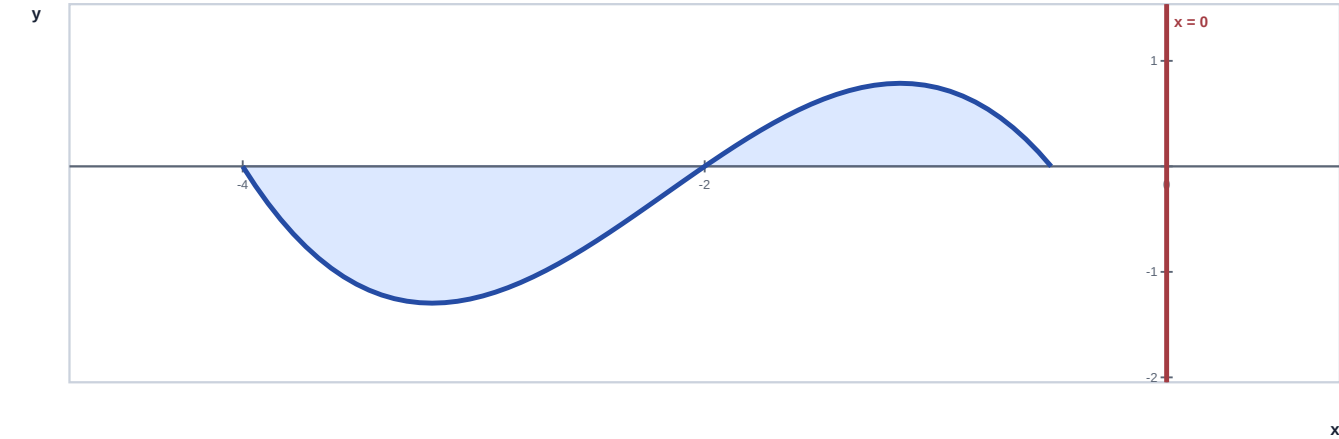
Volumes Using Cylindrical Shells

Use cylindrical shells to find each exact volume. Show radius, positive height, setup, and evaluation.

1. Rotate the shaded region about the vertical line $x = 0$.

$y = \frac{-1}{2} \cdot (x + 4) \cdot (x + 2) \cdot \left(x + \frac{1}{2}\right)$

Region: between this curve and $y = 0$ from $x = -4$ to $-1/2$.

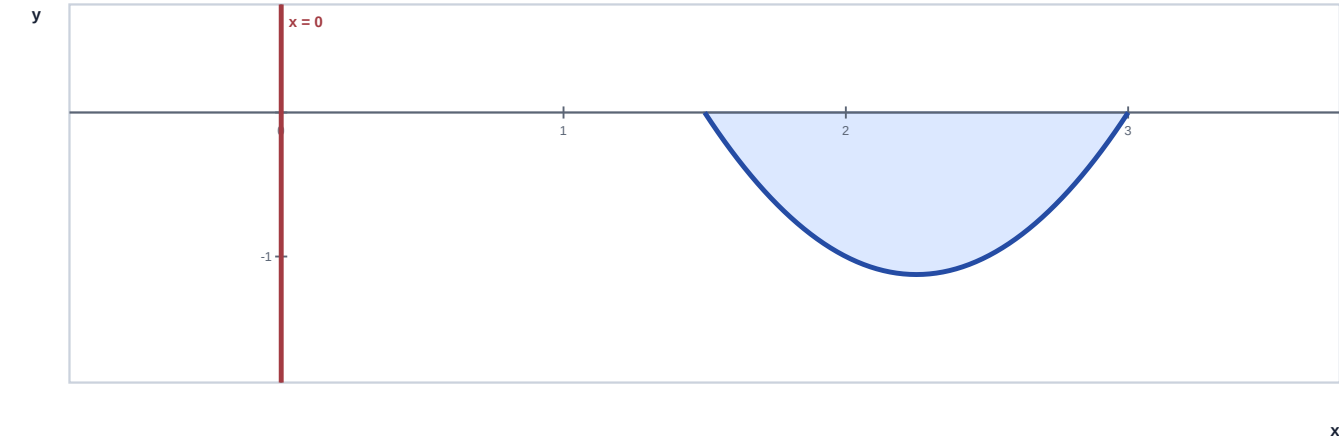


Show shell radius, positive height, setup, and exact volume:

2. Rotate the shaded region about the vertical line $x = 0$.

$y = 2 \cdot \left(x - \frac{3}{2}\right) \cdot (x - 3)$

Region: between this curve and $y = 0$ from $x = 3/2$ to 3 .



Show shell radius, positive height, setup, and exact volume:

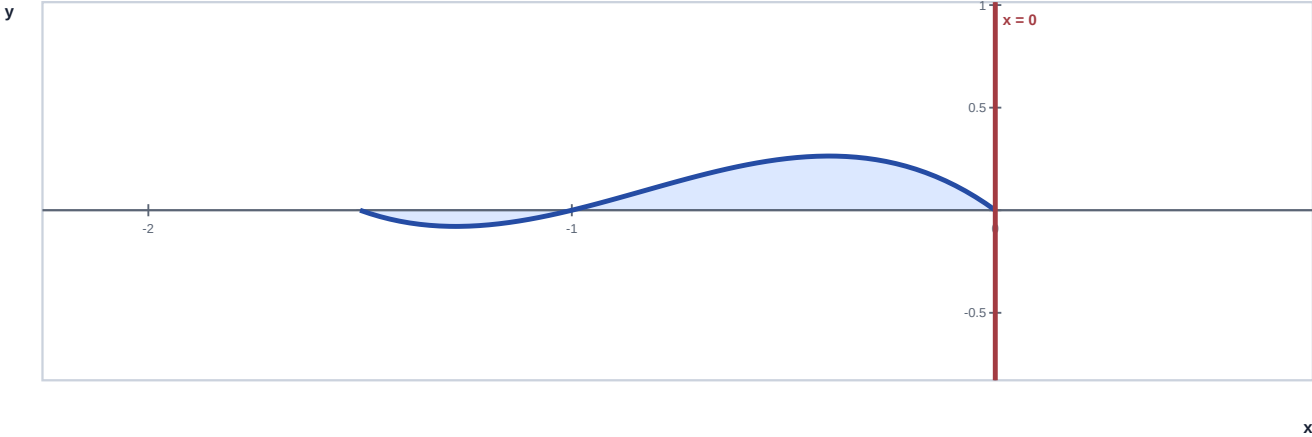
Volumes Using Cylindrical Shells

Use cylindrical shells to find each exact volume. Show radius, positive height, setup, and evaluation.

3. Rotate the shaded region about the vertical line $x = 0$.

$y = -1 \cdot \left(x + \frac{3}{2}\right) \cdot (x + 1) \cdot (x + 0)$

Region: between this curve and $y = 0$ from $x = -3/2$ to 0 .



Show shell radius, positive height, setup, and exact volume:

4. Rotate the shaded region about the vertical line $x = 0$.

$y = \frac{1}{2} \cdot \left(x - \frac{3}{2}\right) \cdot (x - 4)$

Region: between this curve and $y = 0$ from $x = 3/2$ to 4 .



Show shell radius, positive height, setup, and exact volume:

Volumes Using Cylindrical Shells

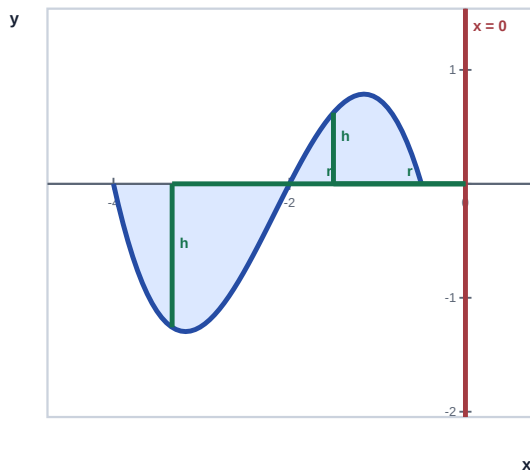
Worked Answer Key

Use cylindrical shells to find each exact volume. Show radius, positive height, setup, and evaluation.

1. $y = F(x)$; rotate about $x = 0$.

$$F(x) = \frac{-1}{2} \cdot (x+4) \cdot (x+2) \cdot \left(x + \frac{1}{2}\right)$$

Roots and limits: -4, -2, -1/2.



$$r(x) = -1 \cdot x$$

Lobe 1: -4 to -2

$$h(x) = \frac{1}{2} \cdot x^3 + \frac{13}{4} \cdot x^2 + \frac{11}{2} \cdot x + 2$$

$$r(x)h(x) = -\frac{1}{2} \cdot x^4 - \frac{13}{4} \cdot x^3 - \frac{11}{2} \cdot x^2 - 2 \cdot x$$

$$2 \cdot \pi \int_{-4}^{-2} -\frac{1}{2} \cdot x^4 - \frac{13}{4} \cdot x^3 - \frac{11}{2} \cdot x^2 - 2 \cdot x \, dx$$

$$P(x) = -\frac{1}{10} \cdot x^5 - \frac{13}{16} \cdot x^4 - \frac{11}{6} \cdot x^3 - x^2$$

$$P(-2) - P(-4) = \frac{13}{15} - \left(\frac{-64}{15} \right)$$

$$V_1 = \frac{154}{15} \cdot \pi$$

Lobe 2: -2 to -1/2

$$h(x) = -\frac{1}{2} \cdot x^3 - \frac{13}{4} \cdot x^2 - \frac{11}{2} \cdot x - 2$$

$$r(x)h(x) = \frac{1}{2} \cdot x^4 + \frac{13}{4} \cdot x^3 + \frac{11}{2} \cdot x^2 + 2 \cdot x$$

$$2 \cdot \pi \int_{-2}^{-1/2} \frac{1}{2} \cdot x^4 + \frac{13}{4} \cdot x^3 + \frac{11}{2} \cdot x^2 + 2 \cdot x \, dx$$

$$P(x) = \frac{1}{10} \cdot x^5 + \frac{13}{16} \cdot x^4 + \frac{11}{6} \cdot x^3 + x^2$$

$$P(-1/2) - P(-2) = \frac{263}{3840} - \left(\frac{-13}{15} \right)$$

$$V_2 = \frac{1197}{640} \cdot \pi$$

$$\text{Total volume} = \frac{154}{15} \cdot \pi + \frac{1197}{640} \cdot \pi$$

$$= \frac{23303}{1920} \cdot \pi$$

cubic units

Volumes Using Cylindrical Shells

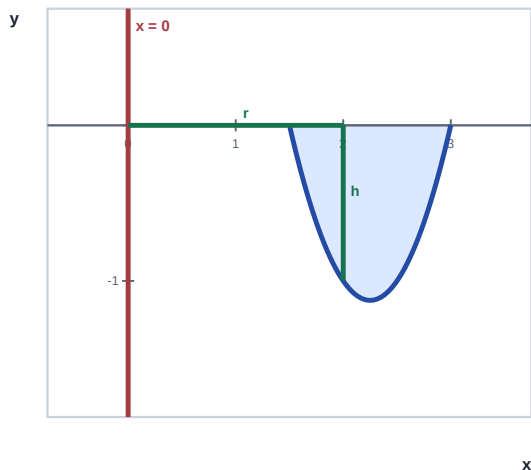
Worked Answer Key

Use cylindrical shells to find each exact volume. Show radius, positive height, setup, and evaluation.

2. $y = F(x)$; rotate about $x = 0$.

$$F(x) = 2 \cdot \left(x - \frac{3}{2} \right) \cdot (x - 3)$$

Roots and limits: $3/2, 3$.



$$r(x) = x$$

Lobe 1: $3/2$ to 3

$$h(x) = -2 \cdot x^2 + 9 \cdot x - 9$$

$$r(x)h(x) = -2 \cdot x^3 + 9 \cdot x^2 - 9 \cdot x$$

$$2 \cdot \pi \int_{\frac{3}{2}}^3 -2 \cdot x^3 + 9 \cdot x^2 - 9 \cdot x \, dx$$

$$P(x) = -\frac{1}{2} \cdot x^4 + 3 \cdot x^3 - \frac{9}{2} \cdot x^2$$

$$P(3) - P(3/2) = 0 - \left(\frac{-81}{32} \right)$$

$$V_1 = \frac{81}{16} \cdot \pi$$

$$\text{Total volume} = \frac{81}{16} \cdot \pi$$

$$= \frac{81}{16} \cdot \pi$$

cubic units

Volumes Using Cylindrical Shells

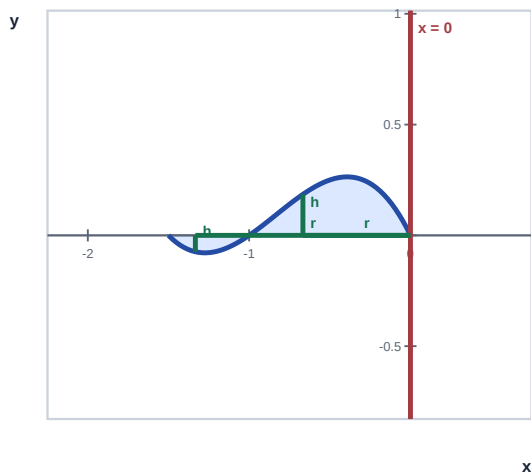
Worked Answer Key

Use cylindrical shells to find each exact volume. Show radius, positive height, setup, and evaluation.

3. $y = F(x)$; rotate about $x = 0$.

$$F(x) = -1 \cdot \left(x + \frac{3}{2}\right) \cdot (x + 1) \cdot (x + 0)$$

Roots and limits: $-3/2, -1, 0$.



$$r(x) = -1 \cdot x$$

Lobe 1: $-3/2$ to -1

$$h(x) = x^3 + \frac{5}{2} \cdot x^2 + \frac{3}{2} \cdot x$$

$$r(x)h(x) = -x^4 - \frac{5}{2} \cdot x^3 - \frac{3}{2} \cdot x^2$$

$$2 \cdot \pi \int_{-3/2}^{-1} -x^4 - \frac{5}{2} \cdot x^3 - \frac{3}{2} \cdot x^2 dx$$

$$P(x) = -\frac{1}{5} \cdot x^5 - \frac{5}{8} \cdot x^4 - \frac{1}{2} \cdot x^3$$

$$P(-1) - P(-3/2) = \frac{3}{40} - \left(\frac{27}{640}\right)$$

$$V_1 = \frac{21}{320} \cdot \pi$$

Lobe 2: -1 to 0

$$h(x) = -x^3 - \frac{5}{2} \cdot x^2 - \frac{3}{2} \cdot x$$

$$r(x)h(x) = x^4 + \frac{5}{2} \cdot x^3 + \frac{3}{2} \cdot x^2$$

$$2 \cdot \pi \int_{-1}^0 x^4 + \frac{5}{2} \cdot x^3 + \frac{3}{2} \cdot x^2 dx$$

$$P(x) = \frac{1}{5} \cdot x^5 + \frac{5}{8} \cdot x^4 + \frac{1}{2} \cdot x^3$$

$$P(0) - P(-1) = 0 - \left(\frac{-3}{40}\right)$$

$$V_2 = \frac{3}{20} \cdot \pi$$

$$\text{Total volume} = \frac{21}{320} \cdot \pi + \frac{3}{20} \cdot \pi$$

$$= \frac{69}{320} \cdot \pi$$

cubic units

Volumes Using Cylindrical Shells

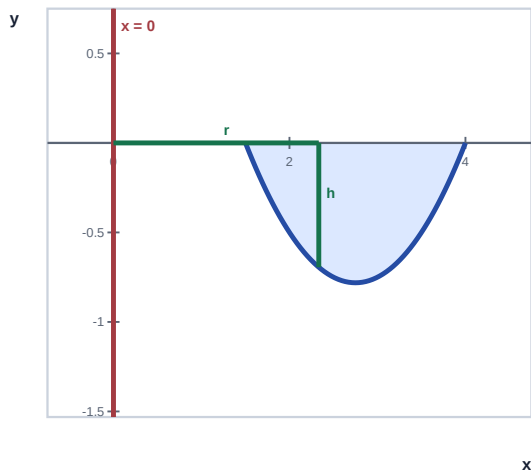
Worked Answer Key

Use cylindrical shells to find each exact volume. Show radius, positive height, setup, and evaluation.

4. $y = F(x)$; rotate about $x = 0$.

$$F(x) = \frac{1}{2} \cdot \left(x - \frac{3}{2} \right) \cdot \left(x - 4 \right)$$

Roots and limits: $3/2$, 4 .



$$r(x) = x$$

Lobe 1: $3/2$ to 4

$$h(x) = -\frac{1}{2} \cdot x^2 + \frac{11}{4} \cdot x - 3$$

$$r(x)h(x) = -\frac{1}{2} \cdot x^3 + \frac{11}{4} \cdot x^2 - 3 \cdot x$$

$$2 \cdot \pi \int_{\frac{3}{2}}^4 -\frac{1}{2} \cdot x^3 + \frac{11}{4} \cdot x^2 - 3 \cdot x \, dx$$

$$P(x) = -\frac{1}{8} \cdot x^4 + \frac{11}{12} \cdot x^3 - \frac{3}{2} \cdot x^2$$

$$P(4) - P(3/2) = \frac{8}{3} - \left(\frac{-117}{128} \right)$$

$$V_1 = \frac{1375}{192} \cdot \pi$$

$$\text{Total volume} = \frac{1375}{192} \cdot \pi$$

$$= \frac{1375}{192} \cdot \pi$$

cubic units