

Volume Using Cross Sections

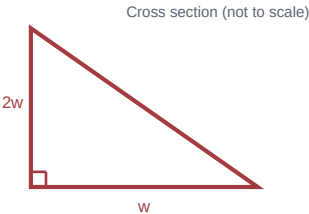
Sketch the base and cross section, then find the exact volume.

1. The base is the full noncircular ellipse inside the measured boundary

$$\frac{(x+3)^2}{25} + \frac{y^2}{16} = 1$$

Cross sections perpendicular to the x-axis are right triangles with perpendicular legs w and 2w.

Sketch the base and a sample slice.



Work:

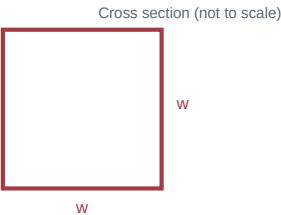
V = _____ cubic units

2. The base is the upper half of a noncircular ellipse inside the measured boundary

$$\frac{\left(x+\frac{3}{2}\right)^2}{9} + \frac{(y+3)^2}{36} = 1$$

Restrict to y >= -3. Cross sections perpendicular to the x-axis are squares with side w.

Sketch the base and a sample slice.



Work:

V = _____ cubic units

Volume Using Cross Sections

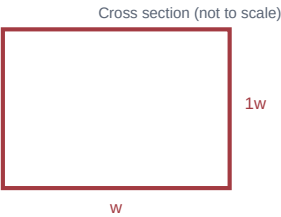
Sketch the base and cross section, then find the exact volume.

3. The base is the upper half-disk inside the measured boundary

$$\frac{(x+1)^2}{36} + \frac{\left(y - \frac{5}{2}\right)^2}{36} = 1$$

Restrict to $y \geq 5/2$. Cross sections perpendicular to the x-axis are rectangles with base w and height 1w.

Sketch the base and a sample slice.



Work:

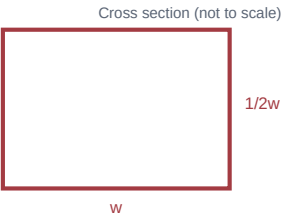
V = _____ cubic units

4. The base is the upper half of a noncircular ellipse inside the measured boundary

$$\frac{\left(x + \frac{1}{2}\right)^2}{36} + \frac{\left(y + \frac{1}{2}\right)^2}{9} = 1$$

Restrict to $y \geq -1/2$. Cross sections perpendicular to the x-axis are rectangles with base w and height 1/2w.

Sketch the base and a sample slice.



Work:

V = _____ cubic units

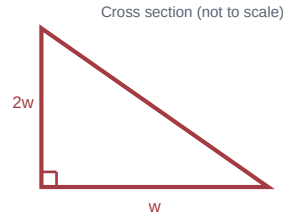
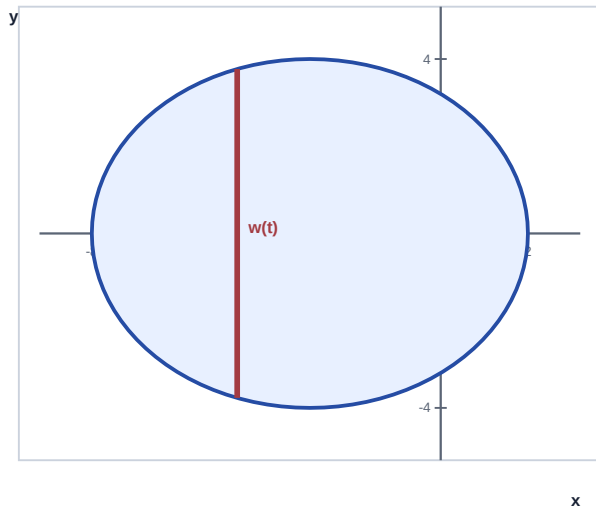
Volume Using Cross Sections

Worked Answer Key

Sketch the base and cross section, then find the exact volume.

1. The base is the full noncircular ellipse inside the measured boundary

$$\frac{(x+3)^2}{25} + \frac{y^2}{16} = 1$$

Cross sections perpendicular to the x-axis are right triangles with perpendicular legs w and $2w$.Bounds: $-8 \leq x \leq 2$

$$R(x) = -\frac{1}{25}x^2 - \frac{6}{25}x + \frac{16}{25}$$

$$w(x) = 8 \cdot \sqrt{-\frac{1}{25}x^2 - \frac{6}{25}x + \frac{16}{25}}$$

$$w(x)^2 = -\frac{64}{25}x^2 - \frac{384}{25}x + \frac{1024}{25}$$

$$A(x) = 1w(x)^2 = -\frac{64}{25}x^2 - \frac{384}{25}x + \frac{1024}{25}$$

$$V = \int_{-8}^2 \left(-\frac{64}{25}x^2 - \frac{384}{25}x + \frac{1024}{25} \right) dx$$

$$P(x) = -\frac{64}{75}x^3 - \frac{192}{25}x^2 + \frac{1024}{25}x$$

$$V = P(2) - P(-8)$$

$$V = \frac{3328}{75} - \left(-\frac{28672}{75} \right) = \frac{1280}{3} \quad \text{cubic units}$$

Volume is the integral of cross-sectional area.

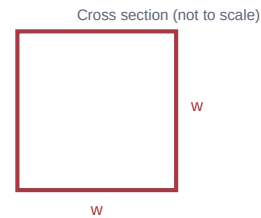
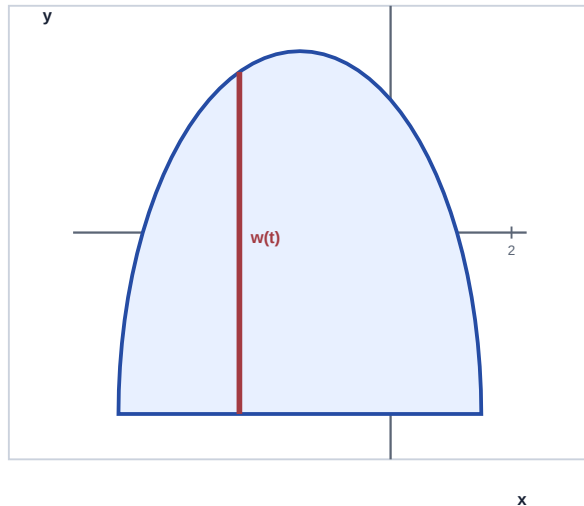
Volume Using Cross Sections

Worked Answer Key

Sketch the base and cross section, then find the exact volume.

2. The base is the upper half of a noncircular ellipse inside the measured boundary

$$\frac{\left(x + \frac{3}{2}\right)^2}{9} + \frac{(y + 3)^2}{36} = 1$$

Restrict to $y \geq -3$. Cross sections perpendicular to the x-axis are squares with side w .Bounds: $-9/2 \leq x \leq 3/2$

$$R(x) = -\frac{1}{9}x^2 - \frac{1}{3}x + \frac{3}{4}$$

$$w(x) = 6\sqrt{-\frac{1}{9}x^2 - \frac{1}{3}x + \frac{3}{4}}$$

$$w(x)^2 = -\frac{4}{9}x^2 - \frac{4}{3}x + 3$$

$$A(x) = 1w(x)^2 = -\frac{4}{9}x^2 - \frac{4}{3}x + 3$$

$$V = \int_{-9/2}^{3/2} \left(-\frac{4}{9}x^2 - \frac{4}{3}x + 3\right) dx$$

$$P(x) = -\frac{4}{27}x^3 - \frac{2}{3}x^2 + 3x$$

$$V = P(3/2) - P(-9/2)$$

$$V = \frac{45}{2} - \left(-\frac{243}{2}\right) = 144$$

cubic units

Volume is the integral of cross-sectional area.

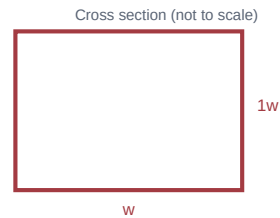
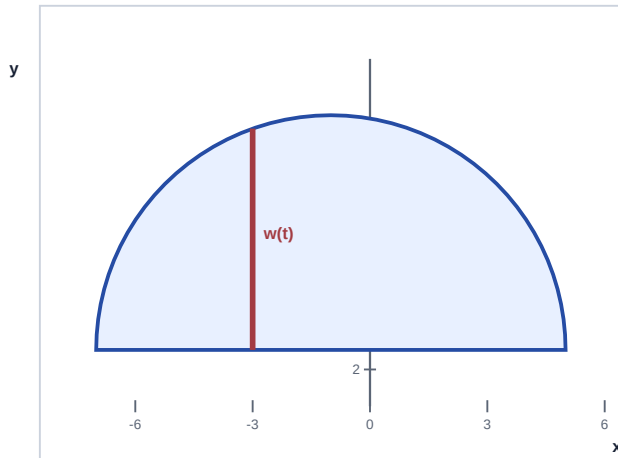
Volume Using Cross Sections

Worked Answer Key

Sketch the base and cross section, then find the exact volume.

3. The base is the upper half-disk inside the measured boundary

$$\frac{(x+1)^2}{36} + \frac{\left(y - \frac{5}{2}\right)^2}{36} = 1$$

Restrict to $y \geq 5/2$. Cross sections perpendicular to the x-axis are rectangles with base w and height $1w$.Bounds: $-7 \leq x \leq 5$

$$R(x) = -\frac{1}{36}x^2 - \frac{1}{18}x + \frac{35}{36}$$

$$w(x) = 6\sqrt{-\frac{1}{36}x^2 - \frac{1}{18}x + \frac{35}{36}}$$

$$w(x)^2 = -x^2 - 2x + 35$$

$$A(x) = 1w(x)^2 = -x^2 - 2x + 35$$

$$V = \int_{-7}^5 (-x^2 - 2x + 35) dx$$

$$P(x) = -\frac{1}{3}x^3 - x^2 + 35x$$

$$V = P(5) - P(-7)$$

$$V = \frac{325}{3} - \left(-\frac{539}{3}\right) = 288 \quad \text{cubic units}$$

Volume is the integral of cross-sectional area.

Volume Using Cross Sections

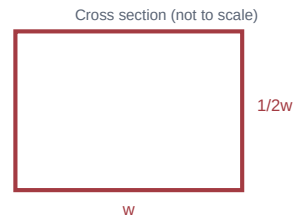
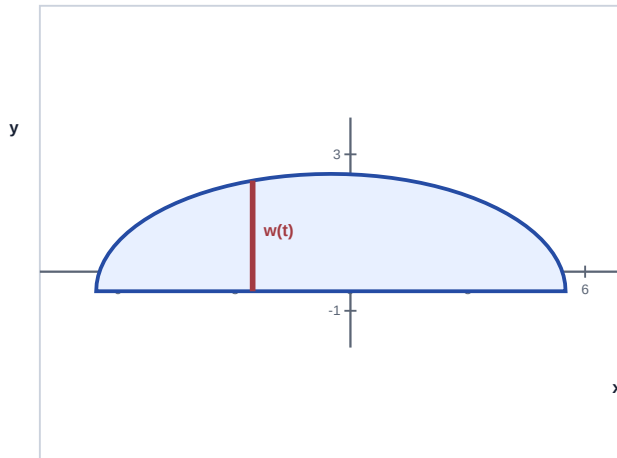
Worked Answer Key

Sketch the base and cross section, then find the exact volume.

4. The base is the upper half of a noncircular ellipse inside the measured boundary

$$\frac{\left(x + \frac{1}{2}\right)^2}{36} + \frac{\left(y + \frac{1}{2}\right)^2}{9} = 1$$

Restrict to $y \geq -1/2$. Cross sections perpendicular to the x-axis are rectangles with base w and height $1/2w$.



Bounds: $-13/2 \leq x \leq 11/2$

$$R(x) = -\frac{1}{36}x^2 - \frac{1}{36}x + \frac{143}{144}$$

$$w(x) = 3\sqrt{-\frac{1}{36}x^2 - \frac{1}{36}x + \frac{143}{144}}$$

$$w(x)^2 = -\frac{1}{4}x^2 - \frac{1}{4}x + \frac{143}{16}$$

$$A(x) = \frac{1}{2}w(x)^2 = -\frac{1}{8}x^2 - \frac{1}{8}x + \frac{143}{32}$$

$$V = \int_{-13/2}^{11/2} \left(-\frac{1}{8}x^2 - \frac{1}{8}x + \frac{143}{32} \right) dx$$

$$P(x) = -\frac{1}{24}x^3 - \frac{1}{16}x^2 + \frac{143}{32}x$$

$$V = P(11/2) - P(-13/2)$$

$$V = \frac{3025}{192} - \left(-\frac{3887}{192} \right) = 36 \quad \text{cubic units}$$

Volume is the integral of cross-sectional area.