

Understanding Motion

Analyze position, velocity, and acceleration on the stated closed time interval.

1. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = 4 - 10t + t^2$$

Set-up and work:



2. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 8$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = -6 - 6t + 6t^2 - 2t^3$$

Set-up and work:



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3. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = 7 - 48t + 18t^2 - 2t^3$$

Set-up and work:



4. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 8$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = -3 - 6t + 6t^2 - 2t^3$$

Set-up and work:



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5. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = 7 + 108 \cdot t - 27 \cdot t^2 + 2 \cdot t^3$$

Set-up and work:



6. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = -4 \cdot t + t^2$$

Set-up and work:



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7. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = -6 + 4 \cdot t - t^2$$

Set-up and work:



8. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = -20 \cdot t + 2 \cdot t^2$$

Set-up and work:



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9. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = 90 \cdot t - 24 \cdot t^2 + 2 \cdot t^3$$

Set-up and work:



10. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 6$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = 4 - 2 \cdot t + t^2$$

Set-up and work:



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11. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 6$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = 4 - 6t + 6t^2 - 2t^3$$

Set-up and work:



12. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

$$s(t) = 7 - 4t + t^2$$

Set-up and work:



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Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

1. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = 4 - 10t + t^2$$

Velocity:

$$v(t) = -10 + 2t$$

Acceleration:

$$a(t) = 2$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=5$ s. Velocity signs: (0, 5) is negative; (5, 12) is positive.

Endpoint positions:

$$\text{displacement} = 24$$

$$s(0)=4 \text{ m and } s(12)=28 \text{ m.}$$

Candidate velocities: $v(0) = -10$ m/s, $v(12) = 14$ m/s. Maximum velocity: 14 m/s at $t = 12$ s.

Rest instants: 5 s. Direction changes: 5 s.

Moving right: (5, 12]. Moving left: [0, 5).

Displacement: $s(12) - s(0) = 24$ m.

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Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

2. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 8$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = -6 - 6t + 6t^2 - 2t^3$$

Velocity:

$$v(t) = -6 + 12t - 6t^2$$

Acceleration:

$$a(t) = 12 - 12t$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=1$ s. Velocity signs: $(0, 1)$ is negative; $(1, 8)$ is negative.

Endpoint positions:

$$\text{displacement} = -688$$

$$s(0) = -6 \text{ m and } s(8) = -694 \text{ m.}$$

Candidate velocities: $v(0) = -6$ m/s, $v(8) = -294$ m/s, $v(1) = 0$ m/s. Maximum velocity: 0 m/s at $t = 1$ s.

Rest instants: 1 s. Direction changes: none; repeated velocity roots do not change sign.

Moving right: never. Moving left: $[0, 1)$ or $(1, 8]$.

Displacement: $s(8) - s(0) = -688$ m.

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Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

3. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = 7 - 48t + 18t^2 - 2t^3$$

Velocity:

$$v(t) = -48 + 36t - 6t^2$$

Acceleration:

$$a(t) = 36 - 12t$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=2$ s, $t=4$ s. Velocity signs: $(0, 2)$ is negative; $(2, 4)$ is positive; $(4, 12)$ is negative.

Endpoint positions:

$$\text{displacement} = -1440$$

$$s(0)=7 \text{ m and } s(12)=-1433 \text{ m.}$$

Candidate velocities: $v(0) = -48$ m/s, $v(12) = -480$ m/s, $v(3) = 6$ m/s. Maximum velocity: 6 m/s at $t = 3$ s.

Rest instants: 2 s, 4 s. Direction changes: 2 s, 4 s.

Moving right: $(2, 4)$. Moving left: $[0, 2)$ or $(4, 12]$.

Displacement: $s(12) - s(0) = -1440$ m.

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Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

4. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 8$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = -3 - 6t + 6t^2 - 2t^3$$

Velocity:

$$v(t) = -6 + 12t - 6t^2$$

Acceleration:

$$a(t) = 12 - 12t$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=1$ s. Velocity signs: $(0, 1)$ is negative; $(1, 8)$ is negative.

Endpoint positions:

$$\text{displacement} = -688$$

$$s(0) = -3 \text{ m and } s(8) = -691 \text{ m.}$$

Candidate velocities: $v(0) = -6$ m/s, $v(8) = -294$ m/s, $v(1) = 0$ m/s. Maximum velocity: 0 m/s at $t = 1$ s.

Rest instants: 1 s. Direction changes: none; repeated velocity roots do not change sign.

Moving right: never. Moving left: $[0, 1)$ or $(1, 8]$.

Displacement: $s(8) - s(0) = -688$ m.

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Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

5. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = 7 + 108 \cdot t - 27 \cdot t^2 + 2 \cdot t^3$$

Velocity:

$$v(t) = 108 - 54 \cdot t + 6 \cdot t^2$$

Acceleration:

$$a(t) = -54 + 12 \cdot t$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=3$ s, $t=6$ s. Velocity signs: $(0, 3)$ is positive; $(3, 6)$ is negative; $(6, 10)$ is positive.

Endpoint positions:

$$\text{displacement} = 380$$

$$s(0)=7 \text{ m and } s(10)=387 \text{ m.}$$

Candidate velocities: $v(0) = 108$ m/s, $v(10) = 168$ m/s, $v(4.5) = -13.5$ m/s. Maximum velocity: 168 m/s at $t = 10$ s.

Rest instants: 3 s, 6 s. Direction changes: 3 s, 6 s.

Moving right: $[0, 3)$ or $(6, 10]$. Moving left: $(3, 6)$.

Displacement: $s(10) - s(0) = 380$ m.

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Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

6. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = -4t + t^2$$

Velocity:

$$v(t) = -4 + 2t$$

Acceleration:

$$a(t) = 2$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=2$ s. Velocity signs: $(0, 2)$ is negative; $(2, 10)$ is positive.

Endpoint positions:

$$\text{displacement} = 60$$

$$s(0)=0 \text{ m and } s(10)=60 \text{ m.}$$

Candidate velocities: $v(0) = -4$ m/s, $v(10) = 16$ m/s. Maximum velocity: 16 m/s at $t = 10$ s.

Rest instants: 2 s. Direction changes: 2 s.

Moving right: $(2, 10]$. Moving left: $[0, 2)$.

Displacement: $s(10) - s(0) = 60$ m.

Understanding Motion

Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

7. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = -6 + 4t - t^2$$

Velocity:

$$v(t) = 4 - 2t$$

Acceleration:

$$a(t) = -2$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=2$ s. Velocity signs: $(0, 2)$ is positive; $(2, 10)$ is negative.

Endpoint positions:

$$\text{displacement} = -60$$

$$s(0) = -6 \text{ m and } s(10) = -66 \text{ m.}$$

Candidate velocities: $v(0) = 4$ m/s, $v(10) = -16$ m/s. Maximum velocity: 4 m/s at $t = 0$ s.

Rest instants: 2 s. Direction changes: 2 s.

Moving right: $[0, 2)$. Moving left: $(2, 10]$.

Displacement: $s(10) - s(0) = -60$ m.

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Analyze position, velocity, and acceleration on the stated closed time interval.

8. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = -20 \cdot t + 2 \cdot t^2$$

Velocity:

$$v(t) = -20 + 4 \cdot t$$

Acceleration:

$$a(t) = 4$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=5$ s. Velocity signs: (0, 5) is negative; (5, 12) is positive.

Endpoint positions:

$$\text{displacement} = 48$$

$$s(0)=0 \text{ m and } s(12)=48 \text{ m.}$$

Candidate velocities: $v(0) = -20$ m/s, $v(12) = 28$ m/s. Maximum velocity: 28 m/s at $t = 12$ s.

Rest instants: 5 s. Direction changes: 5 s.

Moving right: (5, 12]. Moving left: [0, 5).

Displacement: $s(12) - s(0) = 48$ m.

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9. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 12$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = 90 \cdot t - 24 \cdot t^2 + 2 \cdot t^3$$

Velocity:

$$v(t) = 90 - 48 \cdot t + 6 \cdot t^2$$

Acceleration:

$$a(t) = -48 + 12 \cdot t$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=3$ s, $t=5$ s. Velocity signs: $(0, 3)$ is positive; $(3, 5)$ is negative; $(5, 12)$ is positive.

Endpoint positions:

$$\text{displacement} = 1080$$

$$s(0)=0 \text{ m and } s(12)=1080 \text{ m.}$$

Candidate velocities: $v(0) = 90$ m/s, $v(12) = 378$ m/s, $v(4) = -6$ m/s. Maximum velocity: 378 m/s at $t = 12$ s.

Rest instants: 3 s, 5 s. Direction changes: 3 s, 5 s.

Moving right: $[0, 3)$ or $(5, 12]$. Moving left: $(3, 5)$.

Displacement: $s(12) - s(0) = 1080$ m.

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10. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 6$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = 4 - 2 \cdot t + t^2$$

Velocity:

$$v(t) = -2 + 2 \cdot t$$

Acceleration:

$$a(t) = 2$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=1$ s. Velocity signs: (0, 1) is negative; (1, 6) is positive.

Endpoint positions:

$$\text{displacement} = 24$$

$$s(0)=4 \text{ m and } s(6)=28 \text{ m.}$$

Candidate velocities: $v(0) = -2$ m/s, $v(6) = 10$ m/s. Maximum velocity: 10 m/s at $t = 6$ s.

Rest instants: 1 s. Direction changes: 1 s.

Moving right: (1, 6]. Moving left: [0, 1).

Displacement: $s(6) - s(0) = 24$ m.

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Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

11. degree 3

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 6$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = 4 - 6t + 6t^2 - 2t^3$$

Velocity:

$$v(t) = -6 + 12t - 6t^2$$

Acceleration:

$$a(t) = 12 - 12t$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=1$ s. Velocity signs: $(0, 1)$ is negative; $(1, 6)$ is negative.

Endpoint positions:

$$\text{displacement} = -252$$

$$s(0)=4 \text{ m and } s(6)=-248 \text{ m.}$$

Candidate velocities: $v(0) = -6$ m/s, $v(6) = -150$ m/s, $v(1) = 0$ m/s. Maximum velocity: 0 m/s at $t = 1$ s.

Rest instants: 1 s. Direction changes: none; repeated velocity roots do not change sign.

Moving right: never. Moving left: $[0, 1)$ or $(1, 6]$.

Displacement: $s(6) - s(0) = -252$ m.

Understanding Motion

Worked Answer Key

Analyze position, velocity, and acceleration on the stated closed time interval.

12. degree 2

A particle has position $s(t)$ in meters at time t seconds on $0 \leq t \leq 10$. Find: maximum velocity; when direction changes; when it moves left and right; displacement over the interval.

Position:

$$s(t) = 7 - 4t + t^2$$

Velocity:

$$v(t) = -4 + 2t$$

Acceleration:

$$a(t) = 2$$

Rest instants:

$$v(t) = 0$$

Roots in the domain: $t=2$ s. Velocity signs: $(0, 2)$ is negative; $(2, 10)$ is positive.

Endpoint positions:

$$\text{displacement} = 60$$

$$s(0)=7 \text{ m and } s(10)=67 \text{ m.}$$

Candidate velocities: $v(0) = -4$ m/s, $v(10) = 16$ m/s. Maximum velocity: 16 m/s at $t = 10$ s.

Rest instants: 2 s. Direction changes: 2 s.

Moving right: $(2, 10]$. Moving left: $[0, 2)$.

Displacement: $s(10) - s(0) = 60$ m.