

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Trigonometric Substitution Integrals

Choose the standard substitution, state its domain, simplify the radical, and back-substitute to x . Give exact values for definite integrals.

1. Evaluate exactly: $\int \sqrt{x^2 - 9} \, dx$. Use $x = 3 \sec(\theta)$; $x \geq 3$; $0 \leq \theta < \pi/2$.

2. Evaluate exactly: $\int_0^2 \sqrt{9 + x^2} \, dx$. Use $x = 3 \tan(\theta)$; all real x ; $-\pi/2 < \theta < \pi/2$.

Trigonometric Substitution Integrals

Choose the standard substitution, state its domain, simplify the radical, and back-substitute to x. Give exact values for definite integrals.

3. Evaluate exactly: $\int \sqrt{16 - x^2} \, dx$. Use $x = 4 \sin(\theta)$; $|x| \leq 4$; $-\pi/2 \leq \theta \leq \pi/2$.

4. Evaluate exactly: $\int_0^3 2\sqrt{9 - x^2} \, dx$. Use $x = 3 \sin(\theta)$; $|x| \leq 3$; $-\pi/2 \leq \theta \leq \pi/2$.

Trigonometric Substitution Integrals

Choose the standard substitution, state its domain, simplify the radical, and back-substitute to x. Give exact values for definite integrals.

5. Evaluate exactly: integral 3 sqrt(64 + x^2) dx. Use x = 8 tan(theta); all real x; -pi/2 < theta < pi/2.

6. Evaluate exactly: integral from 8 to 8sqrt(2) 3 sqrt(x^2 - 64) dx. Use x = 8 sec(theta); x >= 8; 0 <= theta < pi/2.

Trigonometric Substitution Integrals

Worked Answer Key

Choose the standard substitution, state its domain, simplify the radical, and back-substitute to x . Give exact values for definite integrals.

1. Evaluate exactly: $\int \sqrt{x^2 - 9} \, dx$. Use $x = 3 \sec(\theta)$; $x \geq 3$; $0 \leq \theta < \pi/2$.

Answer: $\frac{1}{2}[x \sqrt{x^2 - 9} - 9 \ln|x + \sqrt{x^2 - 9}|] + C$

- Let $x = 3 \sec(\theta)$; $dx = 3 \sec(\theta) \tan(\theta) \, d\theta$. The domain $x \geq 3$; $0 \leq \theta < \pi/2$ controls the radical sign.
- After simplifying the radical: $9 \int \sec(\theta) \tan^2(\theta) \, d\theta$.
- Integrate in θ and back-substitute: $\frac{1}{2}[x \sqrt{x^2 - 9} - 9 \ln|x + \sqrt{x^2 - 9}|] + C$.

2. Evaluate exactly: $\int_0^3 2 \sqrt{9 + x^2} \, dx$. Use $x = 3 \tan(\theta)$; all real x ; $-\pi/2 < \theta < \pi/2$.

Answer: $9[\sqrt{2} + \ln(1 + \sqrt{2})]$

- Let $x = 3 \tan(\theta)$; $dx = 3 \sec^2(\theta) \, d\theta$. The domain all real x ; $-\pi/2 < \theta < \pi/2$ controls the radical sign.
- After simplifying the radical: $18 \int \sec^3(\theta) \, d\theta$.
- Transform the bounds and evaluate: $9[\sqrt{2} + \ln(1 + \sqrt{2})]$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Trigonometric Substitution Integrals

Worked Answer Key

Choose the standard substitution, state its domain, simplify the radical, and back-substitute to x . Give exact values for definite integrals.

3. Evaluate exactly: $\int \sqrt{16 - x^2} \, dx$. Use $x = 4 \sin(\theta)$; $|x| \leq 4$; $-\pi/2 \leq \theta \leq \pi/2$.

Answer: $\frac{1}{2}[x \sqrt{16 - x^2} + 16 \arcsin(x/4)] + C$

- Let $x = 4 \sin(\theta)$; $dx = 4 \cos(\theta) \, d\theta$. The domain $|x| \leq 4$; $-\pi/2 \leq \theta \leq \pi/2$ controls the radical sign.
- After simplifying the radical: $16 \int \cos(\theta)^2 \, d\theta$.
- Integrate in θ and back-substitute: $\frac{1}{2}[x \sqrt{16 - x^2} + 16 \arcsin(x/4)] + C$.

4. Evaluate exactly: $\int_0^3 2 \sqrt{9 - x^2} \, dx$. Use $x = 3 \sin(\theta)$; $|x| \leq 3$; $-\pi/2 \leq \theta \leq \pi/2$.

Answer: $9/2 \pi$

- Let $x = 3 \sin(\theta)$; $dx = 3 \cos(\theta) \, d\theta$. The domain $|x| \leq 3$; $-\pi/2 \leq \theta \leq \pi/2$ controls the radical sign.
- After simplifying the radical: $18 \int \cos(\theta)^2 \, d\theta$.
- Transform the bounds and evaluate: $9/2 \pi$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Trigonometric Substitution Integrals

Worked Answer Key

Choose the standard substitution, state its domain, simplify the radical, and back-substitute to x . Give exact values for definite integrals.

5. Evaluate exactly: $\int 3\sqrt{64 + x^2} \, dx$. Use $x = 8 \tan(\theta)$; all real x ; $-\pi/2 < \theta < \pi/2$.

Answer: $\frac{3}{2}[x\sqrt{64 + x^2} + 64 \ln|x + \sqrt{64 + x^2}|] + C$

- Let $x = 8 \tan(\theta)$; $dx = 8 \sec(\theta)^2 d\theta$. The domain all real x ; $-\pi/2 < \theta < \pi/2$ controls the radical sign.
- After simplifying the radical: $192 \int \sec(\theta)^3 d\theta$.
- Integrate in θ and back-substitute: $\frac{3}{2}[x\sqrt{64 + x^2} + 64 \ln|x + \sqrt{64 + x^2}|] + C$.

6. Evaluate exactly: $\int_8^{8\sqrt{2}} 3\sqrt{x^2 - 64} \, dx$. Use $x = 8 \sec(\theta)$; $x \geq 8$; $0 \leq \theta < \pi/2$.

Answer: $96[\sqrt{2} - \ln(1 + \sqrt{2})]$

- Let $x = 8 \sec(\theta)$; $dx = 8 \sec(\theta)\tan(\theta) d\theta$. The domain $x \geq 8$; $0 \leq \theta < \pi/2$ controls the radical sign.
- After simplifying the radical: $192 \int \sec(\theta)\tan(\theta)^2 d\theta$.
- Transform the bounds and evaluate: $96[\sqrt{2} - \ln(1 + \sqrt{2})]$.