

Trigonometric Removable Discontinuity Limits

Evaluate each exact limit using standard trigonometric limits and identities.

1.

$$\lim_{x \rightarrow -2} \left(-2 + \frac{1}{2} \cdot \frac{\sin(-3 \cdot (x+2))}{x+2} \right)$$

Exact limit: _____

2.

$$\lim_{x \rightarrow -1} \left(2 + \frac{\sin(x+1)}{\sin(2 \cdot (x+1))} \right)$$

Exact limit: _____

3.

$$\lim_{x \rightarrow 0} \left(1 - \frac{1 - \cos(-1 \cdot x)}{x^2} \right)$$

Exact limit: _____

4.

$$\lim_{x \rightarrow 1} \left(1 - \frac{\cos(-2 \cdot (x-1)) - 1}{\sin(2 \cdot (x-1))^2} \right)$$

Exact limit: _____

5.

$$\lim_{x \rightarrow 2} \left(1 + \frac{1}{2} \cdot \frac{\sin(-1 \cdot (x-2))}{3 \cdot (x-2)} \right)$$

Exact limit: _____

6.

$$\lim_{x \rightarrow \frac{-1}{2} \cdot \pi} \left(-2 - \frac{\sin\left(-2 \cdot \left(x + \frac{1}{2} \cdot \pi\right)\right)}{\sin\left(-3 \cdot \left(x + \frac{1}{2} \cdot \pi\right)\right)} \right)$$

Exact limit: _____

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Evaluate each exact limit using standard trigonometric limits and identities.

7.

$$\lim_{x \rightarrow \frac{1}{2} \cdot \pi} 1 + \frac{1}{2} \cdot \frac{1 - \cos\left(-1 \cdot \left(x - \frac{1}{2} \cdot \pi\right)\right)}{\left(x - \frac{1}{2} \cdot \pi\right)^2}$$

Exact limit: _____

8.

$$\lim_{x \rightarrow -2} 1 + \frac{1}{2} \cdot \frac{\cos\left(-2 \cdot (x + 2)\right) - 1}{\sin\left(3 \cdot (x + 2)\right)^2}$$

Exact limit: _____

Trigonometric Removable Discontinuity Limits

Worked Answer Key

Evaluate each exact limit using standard trigonometric limits and identities.

1.

$$\lim_{x \rightarrow -2} -2 + \frac{1}{2} \cdot \frac{\sin(-3(x+2))}{x+2}$$

Let $h = x - (-2)$, so $h \rightarrow 0$. Use $\sin(t)/t \rightarrow 1$. Use $\sin(t) \rightarrow 0$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is -3. Therefore $-2 + (1/2)(-3) = -7/2$.

$$\text{Limit} = \frac{-7}{2}$$

Original domain: $x + 2 \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/6$.

2.

$$\lim_{x \rightarrow -1} 2 + \frac{\sin(x+1)}{\sin(2(x+1))}$$

Let $h = x - (-1)$, so $h \rightarrow 0$. Split into the two standard sine limits. Use $\sin(t)/t \rightarrow 1$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is $1/2$. Therefore $2 + (1)(1/2) = 5/2$.

$$\text{Limit} = \frac{5}{2}$$

Original domain: $\sin(2(x+1)) \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/4$.

Trigonometric Removable Discontinuity Limits

Worked Answer Key

Evaluate each exact limit using standard trigonometric limits and identities.

3.

$$\lim_{x \rightarrow 0} 1 - \frac{1 - \cos(-1 \cdot x)}{x^2}$$

Let $h = x - (0)$, so $h \rightarrow 0$. Use $1 - \cos(t) = \sin(t)^2 / (1 + \cos(t))$. Use $\sin(t)/t \rightarrow 1$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is $1/2$. Therefore $1 + (-1)(1/2) = -3/2$.

$$\text{Limit} = \frac{-3}{2}$$

Original domain: $x^2 \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/2$.

4.

$$\lim_{x \rightarrow 1} 1 - \frac{\cos(-2 \cdot (x - 1)) - 1}{\sin(2 \cdot (x - 1))^2}$$

Let $h = x - (1)$, so $h \rightarrow 0$. Use $\cos(t) - 1 = -\sin(t)^2 / (1 + \cos(t))$. Use $\sin(t)/t \rightarrow 1$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is $-1/2$. Therefore $1 + (-1)(-1/2) = 3/2$.

$$\text{Limit} = \frac{3}{2}$$

Original domain: $\sin(2 \cdot (x - 1)) \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/4$.

Trigonometric Removable Discontinuity Limits

Worked Answer Key

Evaluate each exact limit using standard trigonometric limits and identities.

5.

$$\lim_{x \rightarrow 2} 1 + \frac{1}{2} \cdot \frac{\sin\left(-1 \cdot (x - 2)\right)}{3 \cdot (x - 2)}$$

Let $h = x - 2$, so $h \rightarrow 0$. Use $\sin(t)/t \rightarrow 1$. Use $\sin(t)/t \rightarrow 1$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is $-1/3$. Therefore $1 + (1/2)(-1/3) = 5/6$.

$$\text{Limit} = \frac{5}{6}$$

Original domain: $3 \cdot (x - 2) \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/6$.

6.

$$\lim_{x \rightarrow \frac{-1}{2} \cdot \pi} -2 - \frac{\sin\left(-2 \cdot \left(x + \frac{1}{2} \cdot \pi\right)\right)}{\sin\left(-3 \cdot \left(x + \frac{1}{2} \cdot \pi\right)\right)}$$

Let $h = x - (-1/2 \cdot \pi)$, so $h \rightarrow 0$. Split into the two standard sine limits. Use $\sin(t)/t \rightarrow 1$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is $2/3$. Therefore $-2 + (-1)(2/3) = -8/3$.

$$\text{Limit} = \frac{-8}{3}$$

Original domain: $\sin\left(-3 \cdot \left(x + \frac{1}{2} \cdot \pi\right)\right) \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/6$.

Trigonometric Removable Discontinuity Limits

Worked Answer Key

Evaluate each exact limit using standard trigonometric limits and identities.

7.

$$\lim_{x \rightarrow \frac{1}{2} \cdot \pi} 1 + \frac{1}{2} \cdot \frac{1 - \cos\left(-1 \cdot \left(x - \frac{1}{2} \cdot \pi\right)\right)}{\left(x - \frac{1}{2} \cdot \pi\right)^2}$$

Let $h = x - (1/2 \cdot \pi)$, so $h \rightarrow 0$. Use $1 - \cos(t) = \sin(t)^2 / (1 + \cos(t))$. Use $\sin(t)/t \rightarrow 1$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is $1/2$. Therefore $1 + (1/2)(1/2) = 5/4$.

Limit = $\frac{5}{4}$

Original domain: $\left(x - \frac{1}{2} \cdot \pi\right)^2 \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/2$.

8.

$$\lim_{x \rightarrow -2} 1 + \frac{1}{2} \cdot \frac{\cos\left(-2 \cdot (x + 2)\right) - 1}{\sin\left(3 \cdot (x + 2)\right)^2}$$

Let $h = x - (-2)$, so $h \rightarrow 0$. Use $\cos(t) - 1 = -\sin(t)^2 / (1 + \cos(t))$. Use $\sin(t)/t \rightarrow 1$ and $\cos(t) \rightarrow 1$ on the sufficient punctured neighborhood. The core limit is $-2/9$. Therefore $1 + (1/2)(-2/9) = 8/9$.

Limit = $\frac{8}{9}$

Original domain: $\sin\left(3 \cdot (x + 2)\right) \neq 0$

A sufficient punctured neighborhood is $0 < |h| < \pi/6$.