Name _____ Score _____ Teacher _____ Date _____

Taylor Series Approximations and Error

Use standard Taylor or Maclaurin series to represent a function, approximate a nearby value, and distinguish the exact target from the polynomial approximation and its error bound.

1. **For f(x)=1/(1-x) centered at a=0, Write the order-4 Taylor polynomial and the infinite-series representation, state its convergence interval, and verify that x=1/10 is inside it. Use the lagrange method for the error statement.**

2. **For f(x)=cos(x) centered at a=0, Write the order-4 Taylor polynomial and the infinite-series representation, state its convergence interval, and verify that x=4/5 is inside it. Use the lagrange method for the error statement.**

Separate the object, test, endpoint, approximation, and conclusion.

Name _____ Score _____ Teacher _____ Date _____

Taylor Series Approximations and Error

Use standard Taylor or Maclaurin series to represent a function, approximate a nearby value, and distinguish the exact target from the polynomial approximation and its error bound.

3. **For f(x)=ln(1+x) centered at a=0, Use the lagrange remainder method to bound the error at the exact point x=2/5. Use the lagrange method for the error statement.**

4. **For f(x)=e^x centered at a=0, Write the order-4 Taylor polynomial and the infinite-series representation, state its convergence interval, and verify that x=9/20 is inside it. Use the lagrange method for the error statement.**

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Taylor Series Approximations and Error

Use standard Taylor or Maclaurin series to represent a function, approximate a nearby value, and distinguish the exact target from the polynomial approximation and its error bound.

5. **For f(x)=cos(x) centered at a=0, Use the order-4 polynomial to approximate the exact target f(2/5)=cos(2/5). Use the alternating method for the error statement.**

6. **For f(x)=sin(x) centered at a=0, Use the lagrange remainder method to bound the error at the exact point x=2/5. Use the lagrange method for the error statement.**

Separate the object, test, endpoint, approximation, and conclusion.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Taylor Series Approximations and Error

Worked Answer Key

Use standard Taylor or Maclaurin series to represent a function, approximate a nearby value, and distinguish the exact target from the polynomial approximation and its error bound.

1. **For f(x)=1/(1-x) centered at a=0, Write the order-4 Taylor polynomial and the infinite-series representation, state its convergence interval, and verify that x=1/10 is inside it. Use the lagrange method for the error statement.**

Answer: P_4(x)=1 +x +x^2 +x^3 +x^4; f(x)=sum_(k=0)^infinity x^k, |x|<1; x=1/10 is admitted.

1. P_4(x)=1 +x +x^2 +x^3 +x^4.
2. The full representation is f(x)=sum_(k=0)^infinity x^k, converging for |x|<1; h=1/10 satisfies this condition.
3. Substitute the exact displacement h=1/10: P_4(1/10) is approximately 1.1111 while the exact target is 1/(1-1/10).
4. On the interval from a to a+h, |f^(5)(t)|<=M with M=5!/(9/10)^6. Lagrange gives |R_4|<=5!/(9/10)^6 * (|1/10|)^5/5!.
5. Using the unrounded polynomial value, the actual error is approximately 0.000011 and is no greater than the exact bound 5!/(9/10)^6 * (|1/10|)^5/5!.

2. **For f(x)=cos(x) centered at a=0, Write the order-4 Taylor polynomial and the infinite-series representation, state its convergence interval, and verify that x=4/5 is inside it. Use the lagrange method for the error statement.**

Answer: P_4(x)=1 -1/2!x^2 +1/4!x^4; f(x)=sum_(j=0)^infinity (-1)^j x^(2j)/(2j)!, all real x; x=4/5 is admitted.

1. P_4(x)=1 -1/2!x^2 +1/4!x^4.
2. The full representation is f(x)=sum_(j=0)^infinity (-1)^j x^(2j)/(2j)!, converging for all real x; h=4/5 satisfies this condition.
3. Substitute the exact displacement h=4/5: P_4(4/5) is approximately 0.697067 while the exact target is cos(4/5).
4. On the interval from a to a+h, |f^(5)(t)|<=M with M=1. Lagrange gives |R_4|<=1 * (|4/5|)^5/5!.
5. Using the unrounded polynomial value, the actual error is approximately 0.00036 and is no greater than the exact bound 1 * (|4/5|)^5/5!.

Separate the object, test, endpoint, approximation, and conclusion.

Name _____ Score _____ Teacher _____ Date _____

Taylor Series Approximations and Error

Worked Answer Key

Use standard Taylor or Maclaurin series to represent a function, approximate a nearby value, and distinguish the exact target from the polynomial approximation and its error bound.

3. **For f(x)=ln(1+x) centered at a=0, Use the lagrange remainder method to bound the error at the exact point x=2/5. Use the lagrange method for the error statement.**

Answer: Exact target ln(1+2/5); P_4(2/5) is approximately 0.334933; |R_4|<=4! * (|2/5|)^5/5!.

1. P_4(x)=1/1x -1/2x^2 +1/3x^3 -1/4x^4.
2. The full representation is f(x)=sum_(k=1)^infinity (-1)^(k+1)x^k/k, converging for -1<x<=1; h=2/5 satisfies this condition.
3. Substitute the exact displacement h=2/5: P_4(2/5) is approximately 0.334933 while the exact target is ln(1+2/5).
4. On the interval from a to a+h, |f^(5)(t)|<=M with M=4!. Lagrange gives |R_4|<=4! * (|2/5|)^5/5!.
5. Using the unrounded polynomial value, the actual error is approximately 0.001539 and is no greater than the exact bound 4! * (|2/5|)^5/5!.

4. **For f(x)=e^x centered at a=0, Write the order-4 Taylor polynomial and the infinite-series representation, state its convergence interval, and verify that x=9/20 is inside it. Use the lagrange method for the error statement.**

Answer: P_4(x)=1 +1/1!x +1/2!x^2 +1/3!x^3 +1/4!x^4; f(x)=sum_(k=0)^infinity x^k/k!, all real x; x=9/20 is admitted.

1. P_4(x)=1 +1/1!x +1/2!x^2 +1/3!x^3 +1/4!x^4.
2. The full representation is f(x)=sum_(k=0)^infinity x^k/k!, converging for all real x; h=9/20 satisfies this condition.
3. Substitute the exact displacement h=9/20: P_4(9/20) is approximately 1.568146 while the exact target is e^(9/20).
4. On the interval from a to a+h, |f^(5)(t)|<=M with M=e^9/20. Lagrange gives |R_4|<=e^9/20 * (|9/20|)^5/5!.
5. Using the unrounded polynomial value, the actual error is approximately 0.000166 and is no greater than the exact bound e^9/20 * (|9/20|)^5/5!.

Name _____ Score _____ Teacher _____ Date _____

Taylor Series Approximations and Error

Worked Answer Key

Use standard Taylor or Maclaurin series to represent a function, approximate a nearby value, and distinguish the exact target from the polynomial approximation and its error bound.

5. **For f(x)=cos(x) centered at a=0, Use the order-4 polynomial to approximate the exact target f(2/5)=cos(2/5). Use the alternating method for the error statement.**

Answer: Exact target cos(2/5); P_4(2/5) is approximately 0.921067; |R_4|<=1/6! * (|2/5|)^6.

1. P_4(x)=1 -1/2!x^2 +1/4!x^4.
2. The full representation is f(x)=sum_(j=0)^infinity (-1)^j x^(2j)/(2j)!, converging for all real x; h=2/5 satisfies this condition.
3. Substitute the exact displacement h=2/5: P_4(2/5) is approximately 0.921067 while the exact target is cos(2/5).
4. At h=2/5, the nonzero terms alternate, their magnitudes decrease, and they approach 0. Thus the alternating remainder is at most the first omitted magnitude 1/6! * (|2/5|)^6.
5. Using the unrounded polynomial value, the actual error is approximately 0.000006 and is no greater than the exact bound 1/6! * (|2/5|)^6.

6. **For f(x)=sin(x) centered at a=0, Use the lagrange remainder method to bound the error at the exact point x=2/5. Use the lagrange method for the error statement.**

Answer: Exact target sin(2/5); P_4(2/5) is approximately 0.389333; |R_4|<=1 * (|2/5|)^5/5!.

1. P_4(x)=1/1!x -1/3!x^3.
2. The full representation is f(x)=sum_(j=0)^infinity (-1)^j x^(2j+1)/(2j+1)!, converging for all real x; h=2/5 satisfies this condition.
3. Substitute the exact displacement h=2/5: P_4(2/5) is approximately 0.389333 while the exact target is sin(2/5).
4. On the interval from a to a+h, |f^(5)(t)|<=M with M=1. Lagrange gives |R_4|<=1 * (|2/5|)^5/5!.
5. Using the unrounded polynomial value, the actual error is approximately 0.000085 and is no greater than the exact bound 1 * (|2/5|)^5/5!.