

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Substitution with Trigonometric Functions

Choose $u = g(x)$, convert the complete differential, integrate the trigonometric form in u , then substitute back and include $+ C$.

1. $\int \frac{-3}{2} \cdot e^{3-x} \cdot \sec(e^{3-x} - 1) \cdot \tan(e^{3-x} - 1) dx$

2. $\int \left(\frac{4}{3} + \frac{4}{3} \cdot x - 4 \cdot x^2 \right) \cdot \cos(2 - 2 \cdot x - x^2 + 2 \cdot x^3) dx$

3. $\int \frac{-1}{2} \cdot \sec(x)^2 dx$

4. $\int \left(-\frac{1}{2} - 2 \cdot x \right) \cdot \csc(-x - 2 \cdot x^2)^2 dx$

5. $\int -6 \cdot x \cdot \csc(-3 - x^2) \cdot \cot(-3 - x^2) dx$

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Worked Answer Key

Choose $u = g(x)$, convert the complete differential, integrate the trigonometric form in u , then substitute back and include $+ C$.

1. Original: $\int \frac{-3}{2} \cdot e^{3-x} \cdot \sec(e^{3-x} - 1) \cdot \tan(e^{3-x} - 1) dx$

$$u = e^{3-x} - 1$$

$$du = (-1 \cdot e^{3-x}) dx$$

$$\left(\frac{-3}{2} \cdot e^{3-x} \right) dx = \frac{3}{2} du$$

$$\text{In } u: \int \frac{3}{2} \cdot \sec(u) \cdot \tan(u) du$$

$$\text{Primitive: } \frac{3}{2} \cdot \sec(u) + C$$

$$\text{Back-substitute: } \frac{3}{2} \cdot \sec(e^{3-x} - 1) + C$$

$$\text{Original domain: } \cos(e^{3-x} - 1) \neq 0$$

The answer holds on each open domain component; constants may differ on disconnected components.

2. Original: $\int \left(\frac{4}{3} + \frac{4}{3} \cdot x - 4 \cdot x^2 \right) \cdot \cos(2 - 2 \cdot x - x^2 + 2 \cdot x^3) dx$

$$u = 2 - 2 \cdot x - x^2 + 2 \cdot x^3$$

$$du = (-2 - 2 \cdot x + 6 \cdot x^2) dx$$

$$\left(\frac{4}{3} + \frac{4}{3} \cdot x - 4 \cdot x^2 \right) dx = \frac{-2}{3} du$$

$$\text{In } u: \int \frac{-2}{3} \cdot \cos(u) du$$

$$\text{Primitive: } -\frac{2}{3} \cdot \sin(u) + C$$

$$\text{Back-substitute: } -\frac{2}{3} \cdot \sin(2 - 2 \cdot x - x^2 + 2 \cdot x^3) + C$$

Original domain: all real x .

The answer holds on each open domain component; constants may differ on disconnected components.

3. Original: $\int \frac{-1}{2} \cdot \sec(x)^2 dx$

$$u = x$$

$$du = (1) dx$$

$$\left(\frac{-1}{2} \right) dx = \frac{-1}{2} du$$

$$\text{In } u: \int \frac{-1}{2} \cdot \sec(u)^2 du$$

$$\text{Primitive: } -\frac{1}{2} \cdot \tan(u) + C$$

$$\text{Back-substitute: } -\frac{1}{2} \cdot \tan(x) + C$$

$$\text{Original domain: } \cos(x) \neq 0$$

The answer holds on each open domain component; constants may differ on disconnected components.

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Choose $u = g(x)$, convert the complete differential, integrate the trigonometric form in u , then substitute back and include $+ C$.

4. Original: $\int \left(-\frac{1}{2} - 2x \right) \csc \left(-x - 2x^2 \right)^2 dx$

$$u = -x - 2x^2$$

$$du = (-1 - 4x) dx$$

$$\left(-\frac{1}{2} - 2x \right) dx = \frac{1}{2} du$$

$$\text{In } u: \int \frac{1}{2} \cdot \csc(u)^2 du$$

$$\text{Primitive: } -\frac{1}{2} \cdot \cot(u) + C$$

$$\text{Back-substitute: } -\frac{1}{2} \cdot \cot(-x - 2x^2) + C$$

$$\text{Original domain: } \sin(-x - 2x^2) \neq 0$$

The answer holds on each open domain component; constants may differ on disconnected components.

5. Original: $\int -6 \cdot x \cdot \csc(-3 - x^2) \cdot \cot(-3 - x^2) dx$

$$u = -3 - x^2$$

$$du = (-2 \cdot x) dx$$

$$(-6 \cdot x) dx = 3 du$$

$$\text{In } u: \int 3 \cdot \csc(u) \cdot \cot(u) du$$

$$\text{Primitive: } -3 \cdot \csc(u) + C$$

$$\text{Back-substitute: } -3 \cdot \csc(-3 - x^2) + C$$

$$\text{Original domain: } \sin(-3 - x^2) \neq 0$$

The answer holds on each open domain component; constants may differ on disconnected components.