

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Substitution with the Power Rule

Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

1.
$$\int (12 \cdot x + 2) \cdot (3 \cdot x^2 + x + 1)^{\frac{1}{2}} dx$$

2.
$$\int -1 \cdot (x - 2)^1 dx$$

3.
$$\int (6 \cdot x + 3) \cdot (3 \cdot x^2 + 3 \cdot x - 3)^{-2} dx$$

4.
$$\int -2 \cdot (-x + 3)^2 dx$$

5.
$$\int \frac{-2}{3} \cdot (-x + 3)^{\frac{3}{2}} dx$$

6.
$$\int -4 \cdot x \cdot (x^2 + 3)^{-3} dx$$

7.
$$\int 6 \cdot (-3 \cdot x + 2)^3 dx$$

8.
$$\int -2 \cdot x \cdot (-2 \cdot x^2 + 3)^{\frac{-1}{2}} dx$$

Substitution with the Power Rule

Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

9. $\int (2 \cdot x + 1) \cdot (2 \cdot x^2 + 2 \cdot x)^{-4} dx$

10. $\int \frac{-2}{3} \cdot (-x - 3)^4 dx$

11. $\int 6 \cdot x \cdot (2 \cdot x^2 + 1)^{\frac{-3}{2}} dx$

12. $\int \frac{-9}{2} \cdot (-3 \cdot x + 1)^{-2} dx$

13. $\int (12 \cdot x + 4) \cdot (-3 \cdot x^2 - 2 \cdot x)^1 dx$

14. $\int (-2 \cdot x - 3)^{-3} dx$

15. $\int \left(-2 \cdot x - \frac{3}{2}\right) \cdot (2 \cdot x^2 + 3 \cdot x - 2)^{\frac{1}{2}} dx$

16. $\int 4 \cdot (2 \cdot x - 1)^2 dx$

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Substitution with the Power Rule

Worked Answer Key

Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

1. Original: $\int (12 \cdot x + 2) (3 \cdot x^2 + x + 1)^{\frac{1}{2}} dx$

$$u = 3 \cdot x^2 + x + 1$$

$$du = (6 \cdot x + 1) dx$$

$$(12 \cdot x + 2) dx = 2 du$$

$$\text{In } u: \int 2 \cdot u^{\frac{1}{2}} du$$

$$\text{Primitive: } \frac{4}{3} \cdot u^{\frac{3}{2}} + C$$

$$\text{Back-substitute: } \frac{4}{3} (3 \cdot x^2 + x + 1)^{\frac{3}{2}} + C$$

Original domain: $g(x) \geq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

2. Original: $\int -1 \cdot (x - 2)^1 dx$

$$u = x - 2$$

$$du = (1) dx$$

$$(-1) dx = -1 du$$

$$\text{In } u: \int -1 \cdot u^1 du$$

$$\text{Primitive: } -\frac{1}{2} \cdot u^2 + C$$

$$\text{Back-substitute: } -\frac{1}{2} (x - 2)^2 + C$$

Original domain: all real x . The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

3. Original: $\int (6 \cdot x + 3) (3 \cdot x^2 + 3 \cdot x - 3)^{-2} dx$

$$u = 3 \cdot x^2 + 3 \cdot x - 3$$

$$du = (6 \cdot x + 3) dx$$

$$(6 \cdot x + 3) dx = 1 du$$

$$\text{In } u: \int u^{-2} du$$

$$\text{Primitive: } -u^{-1} + C$$

$$\text{Back-substitute: } - (3 \cdot x^2 + 3 \cdot x - 3)^{-1} + C$$

Original domain: $g(x) \neq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

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Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

4. Original: $\int -2 \cdot (-x+3)^2 dx$

$$u = -x+3$$

$$du = (-1) dx$$

$$(-2) dx = 2 du$$

$$\text{In } u: \int 2 \cdot u^2 du$$

$$\text{Primitive: } \frac{2}{3} \cdot u^3 + C$$

$$\text{Back-substitute: } \frac{2}{3} \cdot (-x+3)^3 + C$$

Original domain: all real x . The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

5. Original: $\int \frac{-2}{3} \cdot (-x+3)^{\frac{3}{2}} dx$

$$u = -x+3$$

$$du = (-1) dx$$

$$\left(\frac{-2}{3}\right) dx = \frac{2}{3} du$$

$$\text{In } u: \int \frac{2}{3} \cdot u^{\frac{3}{2}} du$$

$$\text{Primitive: } \frac{4}{15} \cdot u^{\frac{5}{2}} + C$$

$$\text{Back-substitute: } \frac{4}{15} \cdot (-x+3)^{\frac{5}{2}} + C$$

Original domain: $g(x) \geq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

6. Original: $\int -4 \cdot x \cdot (x^2+3)^{-3} dx$

$$u = x^2+3$$

$$du = (2 \cdot x) dx$$

$$(-4 \cdot x) dx = -2 du$$

$$\text{In } u: \int -2 \cdot u^{-3} du$$

$$\text{Primitive: } u^{-2} + C$$

$$\text{Back-substitute: } (x^2+3)^{-2} + C$$

Original domain: $g(x) \neq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

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Worked Answer Key

Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

7. Original: $\int 6 \cdot (-3 \cdot x + 2)^3 dx$

$$u = -3 \cdot x + 2$$

$$du = (-3) dx$$

$$(6) dx = -2 du$$

$$\text{In } u: \int -2 \cdot u^3 du$$

$$\text{Primitive: } -\frac{1}{2} \cdot u^4 + C$$

$$\text{Back-substitute: } -\frac{1}{2} \cdot (-3 \cdot x + 2)^4 + C$$

Original domain: all real x . The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

8. Original: $\int -2 \cdot x \cdot (-2 \cdot x^2 + 3)^{-\frac{1}{2}} dx$

$$u = -2 \cdot x^2 + 3$$

$$du = (-4 \cdot x) dx$$

$$(-2 \cdot x) dx = \frac{1}{2} du$$

$$\text{In } u: \int \frac{1}{2} \cdot u^{-\frac{1}{2}} du$$

$$\text{Primitive: } u^{\frac{1}{2}} + C$$

$$\text{Back-substitute: } (-2 \cdot x^2 + 3)^{\frac{1}{2}} + C$$

Original domain: $g(x) > 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

9. Original: $\int (2 \cdot x + 1)(2 \cdot x^2 + 2 \cdot x)^{-4} dx$

$$u = 2 \cdot x^2 + 2 \cdot x$$

$$du = (4 \cdot x + 2) dx$$

$$(2 \cdot x + 1) dx = \frac{1}{2} du$$

$$\text{In } u: \int \frac{1}{2} \cdot u^{-4} du$$

$$\text{Primitive: } -\frac{1}{6} \cdot u^{-3} + C$$

$$\text{Back-substitute: } -\frac{1}{6} \cdot (2 \cdot x^2 + 2 \cdot x)^{-3} + C$$

Original domain: $g(x) \neq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

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Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

10. Original: $\int \frac{-2}{3} (-x - 3)^4 dx$

$$u = -x - 3$$

$$du = (-1) dx$$

$$\left(\frac{-2}{3} \right) dx = \frac{2}{3} du$$

$$\text{In } u: \int \frac{2}{3} \cdot u^4 du$$

$$\text{Primitive: } \frac{2}{15} \cdot u^5 + C$$

$$\text{Back-substitute: } \frac{2}{15} (-x - 3)^5 + C$$

Original domain: all real x . The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

11.

$$\text{Original: } \int 6 \cdot x \cdot (2 \cdot x^2 + 1)^{\frac{-3}{2}} dx$$

$$u = 2 \cdot x^2 + 1$$

$$du = (4 \cdot x) dx$$

$$(6 \cdot x) dx = \frac{3}{2} du$$

$$\text{In } u: \int \frac{3}{2} \cdot u^{\frac{-3}{2}} du$$

$$\text{Primitive: } -3 \cdot u^{\frac{-1}{2}} + C$$

$$\text{Back-substitute: } -3 \cdot (2 \cdot x^2 + 1)^{\frac{-1}{2}} + C$$

Original domain: $g(x) > 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

12. Original: $\int \frac{-9}{2} (-3 \cdot x + 1)^{-2} dx$

$$u = -3 \cdot x + 1$$

$$du = (-3) dx$$

$$\left(\frac{-9}{2} \right) dx = \frac{3}{2} du$$

$$\text{In } u: \int \frac{3}{2} \cdot u^{-2} du$$

$$\text{Primitive: } -\frac{3}{2} \cdot u^{-1} + C$$

$$\text{Back-substitute: } -\frac{3}{2} (-3 \cdot x + 1)^{-1} + C$$

Original domain: $g(x) \neq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

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Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

13. Original: $\int (12 \cdot x + 4) (-3 \cdot x^2 - 2 \cdot x)^1 dx$

$$u = -3 \cdot x^2 - 2 \cdot x$$

$$du = (-6 \cdot x - 2) dx$$

$$(12 \cdot x + 4) dx = -2 du$$

$$\text{In } u: \int -2 \cdot u^1 du$$

$$\text{Primitive: } -u^2 + C$$

$$\text{Back-substitute: } -(-3 \cdot x^2 - 2 \cdot x)^2 + C$$

Original domain: all real x . The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

14. Original: $\int (-2 \cdot x - 3)^{-3} dx$

$$u = -2 \cdot x - 3$$

$$du = (-2) dx$$

$$(1) dx = \frac{-1}{2} du$$

$$\text{In } u: \int \frac{-1}{2} \cdot u^{-3} du$$

$$\text{Primitive: } \frac{1}{4} \cdot u^{-2} + C$$

$$\text{Back-substitute: } \frac{1}{4} \cdot (-2 \cdot x - 3)^{-2} + C$$

Original domain: $g(x) \neq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

15. Original: $\int \left(-2 \cdot x - \frac{3}{2}\right) (2 \cdot x^2 + 3 \cdot x - 2)^{\frac{1}{2}} dx$

$$u = 2 \cdot x^2 + 3 \cdot x - 2$$

$$du = (4 \cdot x + 3) dx$$

$$\left(-2 \cdot x - \frac{3}{2}\right) dx = \frac{-1}{2} du$$

$$\text{In } u: \int \frac{-1}{2} \cdot u^{\frac{1}{2}} du$$

$$\text{Primitive: } -\frac{1}{3} \cdot u^{\frac{3}{2}} + C$$

$$\text{Back-substitute: } -\frac{1}{3} \cdot (2 \cdot x^2 + 3 \cdot x - 2)^{\frac{3}{2}} + C$$

Original domain: $g(x) \geq 0$. The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

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Choose $u = g(x)$, convert the full differential factor, integrate in u , then substitute back and include $+ C$.

16. Original: $\int 4 \cdot (2 \cdot x - 1)^2 dx$

$u = 2 \cdot x - 1$

$du = (2) dx$

$(4) dx = 2 du$

In u : $\int 2 \cdot u^2 du$

Primitive: $\frac{2}{3} \cdot u^3 + C$

Back-substitute: $\frac{2}{3} \cdot (2 \cdot x - 1)^3 + C$

Original domain: all real x . The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.