

Logarithmic and Exponential Substitution

Choose $u = g(x)$, convert the complete differential, integrate in u , then substitute back and include $+ C$.

1.
$$\int \frac{\frac{-4}{3}}{-2 \cdot x} dx$$

$u = -2 \cdot x$

2.
$$\int \frac{3 \cdot e^{-x+2}}{-e^{-x+2}+1} dx$$

$u = -e^{-x+2}+1$

3.
$$\int \frac{3}{2} \cdot e^{3x-1} dx$$

$u = 3x-1$

4.
$$\int \frac{3}{2} \cdot e^{-3x-2} \cdot e^{e^{-3x-2}+1} dx$$

$u = e^{-3x-2}+1$

5.
$$\int \frac{\frac{-1}{2}}{-x-3} \cdot e^{\ln(-x-3)-1} dx$$

$u = \ln(-x-3)-1$

6.
$$\int \frac{-8x+2}{2x^2-x-2} dx$$

$u = 2x^2-x-2$

7.
$$\int \frac{\frac{1}{2}}{-x+3} \cdot \frac{1}{\ln(-x+3)+2} dx$$

$u = \ln(-x+3)+2$

8.
$$\int (2x-2) \cdot e^{x^2-2x+3} dx$$

$u = x^2-2x+3$

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Logarithmic and Exponential Substitution

Worked Answer Key

Choose $u = g(x)$, convert the complete differential, integrate in u , then substitute back and include $+ C$.

1. Original: $\int \frac{-4}{-2-x} dx$

$$u = -2 - x$$

$$du = (-1) dx$$

$$\left(\frac{-4}{-2-x}\right) dx = \frac{4}{1} du$$

$$\ln u: \int \frac{4}{1} du$$

$$\text{Primitive: } 4 \ln(|u|) + C$$

$$\text{Back-substitute: } 4 \ln(|-2-x|) + C$$

$$\text{Domain: } -2-x \neq 0$$

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

2. Original: $\int \frac{3 \cdot e^{-x+2}}{-e^{-x+2}+1} dx$

$$u = -e^{-x+2}+1$$

$$du = (-e^{-x+2}) dx$$

$$\left(3 \cdot e^{-x+2}\right) dx = -3 du$$

$$\ln u: \int -3 \cdot \frac{1}{u} du$$

$$\text{Primitive: } -3 \ln(|u|) + C$$

$$\text{Back-substitute: } -3 \ln(|-e^{-x+2}+1|) + C$$

$$\text{Domain: } -e^{-x+2}+1 \neq 0$$

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

3. Original: $\int \frac{3}{2} \cdot e^{3x-1} dx$

$$u = 3x-1$$

$$du = (3) dx$$

$$\left(\frac{3}{2}\right) dx = \frac{1}{2} du$$

$$\ln u: \int \frac{1}{2} \cdot e^u du$$

$$\text{Primitive: } \frac{1}{2} \cdot e^u + C$$

$$\text{Back-substitute: } \frac{1}{2} \cdot e^{3x-1} + C$$

Original domain: all real x .

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Logarithmic and Exponential Substitution

Worked Answer Key

Choose $u = g(x)$, convert the complete differential, integrate in u , then substitute back and include $+ C$.

4. Original: $\int \frac{3}{2} \cdot e^{-3 \cdot x - 2} \cdot e^{e^{-3 \cdot x - 2} + 1} dx$

$$u = e^{-3 \cdot x - 2} + 1$$

$$du = (-3 \cdot e^{-3 \cdot x - 2}) dx$$

$$\left(\frac{3}{2} \cdot e^{-3 \cdot x - 2} \right) dx = \frac{-1}{2} du$$

$$\text{In } u: \int \frac{-1}{2} \cdot e^u du$$

$$\text{Primitive: } -\frac{1}{2} \cdot e^u + C$$

$$\text{Back-substitute: } -\frac{1}{2} \cdot e^{e^{-3 \cdot x - 2} + 1} + C$$

Original domain: all real x .

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

5. Original: $\int \frac{-1}{-x-3} \cdot e^{\ln(-x-3)-1} dx$

$$u = \ln(-x-3) - 1$$

$$du = \left(\frac{-1}{-x-3} \right) dx$$

$$\left(\frac{-1}{-x-3} \right) dx = \frac{1}{2} du$$

$$\text{In } u: \int \frac{1}{2} \cdot e^u du$$

$$\text{Primitive: } \frac{1}{2} \cdot e^u + C$$

$$\text{Back-substitute: } \frac{1}{2} \cdot e^{\ln(-x-3)-1} + C$$

$$\text{Domain: } -x-3 > 0$$

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

6. Original: $\int \frac{-8 \cdot x + 2}{2 \cdot x^2 - x - 2} dx$

$$u = 2 \cdot x^2 - x - 2$$

$$du = (4 \cdot x - 1) dx$$

$$(-8 \cdot x + 2) dx = -2 du$$

$$\text{In } u: \int -2 \cdot \frac{1}{u} du$$

$$\text{Primitive: } -2 \cdot \ln(|u|) + C$$

$$\text{Back-substitute: } -2 \cdot \ln(|2 \cdot x^2 - x - 2|) + C$$

$$\text{Domain: } 2 \cdot x^2 - x - 2 \neq 0$$

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

Logarithmic and Exponential Substitution

Worked Answer Key

Choose $u = g(x)$, convert the complete differential, integrate in u , then substitute back and include $+ C$.

7.
$$\int \frac{\frac{1}{2}}{\ln(-x+3)+2} dx$$

Original:
$$\int \frac{\frac{1}{2}}{\ln(-x+3)+2} dx$$

$u = \ln(-x+3)+2$

$du = \left(\frac{-1}{-x+3} \right) dx$

$\left(\frac{\frac{1}{2}}{-x+3} \right) dx = \frac{-1}{2} du$

In u : $\int \frac{-1}{2} \cdot \frac{1}{u} du$

Primitive: $-\frac{1}{2} \cdot \ln(|u|) + C$

Back-substitute: $-\frac{1}{2} \cdot \ln(|\ln(-x+3)+2|) + C$

Domain: $-x+3 > 0$

and $\ln(-x+3)+2 \neq 0$

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.

8.
$$\int (2 \cdot x - 2) \cdot e^{x^2 - 2 \cdot x + 3} dx$$

Original:
$$\int (2 \cdot x - 2) \cdot e^{x^2 - 2 \cdot x + 3} dx$$

$u = x^2 - 2 \cdot x + 3$

$du = (2 \cdot x - 2) dx$

$(2 \cdot x - 2) dx = 1 du$

In u : $\int e^u du$

Primitive: $e^u + C$

Back-substitute: $e^{x^2 - 2 \cdot x + 3} + C$

Original domain: all real x .

The antiderivative applies on each open interval within this domain; constants may differ on disconnected intervals.