

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Substitution with Inverse Trigonometric Functions

Choose $u = g(x)$, convert the complete differential, integrate the inverse-trigonometric pattern in u , then substitute back and include $+ C$.

1. Use arccot:

$$\int \frac{\cos(-2-x)}{9+(2\sin(-2-x)-2)^2} dx$$

$$u = 2\sin(-2-x)-2$$

5. Use arctan:

$$\int \frac{\frac{1}{2}+3x}{1+(-1-x-3x^2)^2} dx$$

$$u = -1-x-3x^2$$

2. Use arcsec:

$$\int \frac{4}{|1-2x|\sqrt{(1-2x)^2-4}} dx$$

$$u = 1-2x$$

6. Use arcsin:

$$\int \frac{\frac{4}{3}}{\sqrt{9-(2\ln(-3-x)-1)^2}} dx$$

$$u = 2\ln(-3-x)-1$$

3. Use arccsc:

$$\int \frac{\frac{4}{-3-2x}}{|-2\ln(-3-2x)-1|\sqrt{(-2\ln(-3-2x)-1)^2-9}} dx$$

$$u = -2\ln(-3-2x)-1$$

7. Use arcsin:

$$\int \frac{2+12x-12x^2}{\sqrt{1-(-2-x-3x^2+2x^3)^2}} dx$$

$$u = -2-x-3x^2+2x^3$$

4. Use arccos:

$$\int \frac{-6\sin(2-2x)}{\sqrt{4-(\cos(2-2x)+0)^2}} dx$$

$$u = \cos(2-2x)+0$$

8. Use arccsc:

$$\int \frac{\frac{-3}{2}\sec(-2+3x)^2}{|\tan(-2+3x)-1|\sqrt{(\tan(-2+3x)-1)^2-16}} dx$$

$$u = \tan(-2+3x)-1$$

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Substitution with Inverse Trigonometric Functions

Worked Answer Key

Choose $u = g(x)$, convert the complete differential, integrate the inverse-trigonometric pattern in u , then substitute back and include $+ C$.

1. Original: $\int \frac{\cos(-2-x)}{9 + (2 \cdot \sin(-2-x) - 2)^2} dx$

$u = 2 \cdot \sin(-2-x) - 2$

$du = -2 \cdot \cos(-2-x) \cdot dx$

Scaled: $\cos(-2-x) \cdot dx = \frac{-1}{2} \cdot du$

In u : $\int \frac{\frac{-1}{2}}{9 + u^2} du$

Coefficient: $\frac{\frac{-1}{2}}{-3} = \frac{1}{6}$

Primitive: $\frac{1}{6} \cdot \operatorname{arccot}\left(\frac{u}{3}\right) + C$

Back-substitute: $\frac{1}{6} \cdot \operatorname{arccot}\left(\frac{2 \cdot \sin(-2-x) - 2}{3}\right) + C$

Domain $g(x)$: $2 \cdot \sin(-2-x) - 2$

The inverse kernel permits all real $g(x)$. On each open domain component; constants may differ on disconnected components.

2. Original: $\int \frac{4}{|1-2x| \sqrt{(1-2x)^2 - 4}} dx$

$u = 1-2x$

$du = -2 \cdot dx$

Scaled: $4 \cdot dx = -2 \cdot du$

In u : $\int \frac{-2}{|u| \sqrt{u^2 - 4}} du$

Coefficient: $\frac{-2}{2} = -1$

Primitive: $-\operatorname{arcsec}\left(\frac{u}{2}\right) + C$

Back-substitute: $-\operatorname{arcsec}\left(\frac{1-2x}{2}\right) + C$

Domain $g(x)$: $1-2x$

Inverse kernel: $|1-2x| > 2$

On each open domain component; constants may differ on disconnected components.

Substitution with Inverse Trigonometric Functions

Worked Answer Key

Choose $u = g(x)$, convert the complete differential, integrate the inverse-trigonometric pattern in u , then substitute back and include $+ C$.

3.

$$\text{Original: } \int \frac{4}{|-2 \ln(-3 - 2x) - 1| \sqrt{(-2 \ln(-3 - 2x) - 1)^2 - 9}} dx$$

$$u = -2 \ln(-3 - 2x) - 1$$

$$du = \frac{4}{-3 - 2x} dx$$

$$\text{Scaled: } \frac{4}{-3 - 2x} dx = du$$

$$\ln u: \int \frac{1}{|u| \sqrt{u^2 - 9}} du$$

$$\text{Coefficient: } \frac{1}{-3} = \frac{-1}{3}$$

$$\text{Primitive: } -\frac{1}{3} \cdot \operatorname{arccsc}\left(\frac{u}{3}\right) + C$$

$$\text{Back-substitute: } -\frac{1}{3} \cdot \operatorname{arccsc}\left(\frac{-2 \ln(-3 - 2x) - 1}{3}\right) + C$$

$$\text{Domain } g(x): -2 \ln(-3 - 2x) - 1$$

$$\text{Log: } -3 - 2x > 0$$

$$\text{Inverse kernel: } |-2 \ln(-3 - 2x) - 1| > 3$$

On each open domain component; constants may differ on disconnected components.

4.

$$\text{Original: } \int \frac{-6 \sin(2 - 2x)}{\sqrt{4 - (\cos(2 - 2x) + 0)^2}} dx$$

$$u = \cos(2 - 2x) + 0$$

$$du = -2 \sin(2 - 2x) dx$$

$$\text{Scaled: } -6 \sin(2 - 2x) dx = -3 du$$

$$\ln u: \int \frac{-3}{\sqrt{4 - u^2}} du$$

$$\text{Coefficient: } \frac{-3}{-1} = 3$$

$$\text{Primitive: } 3 \cdot \arccos\left(\frac{u}{2}\right) + C$$

$$\text{Back-substitute: } 3 \cdot \arccos\left(\frac{\cos(2 - 2x) + 0}{2}\right) + C$$

$$\text{Domain } g(x): \cos(2 - 2x) + 0$$

$$\text{Inverse kernel: } |\cos(2 - 2x) + 0| < 2$$

On each open domain component; constants may differ on disconnected components.

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Choose $u = g(x)$, convert the complete differential, integrate the inverse-trigonometric pattern in u , then substitute back and include $+ C$.

5.

$$\text{Original: } \int \frac{\frac{1}{2} + 3 \cdot x}{1 + \left(-1 - x - 3 \cdot x^2\right)^2} dx$$

$$u = -1 - x - 3 \cdot x^2$$

$$du = \left(-1 - 6 \cdot x\right) \cdot dx$$

$$\text{Scaled: } \left(\frac{1}{2} + 3 \cdot x\right) \cdot dx = \frac{-1}{2} \cdot du$$

$$\text{In } u: \int \frac{\frac{-1}{2}}{1 + u^2} du$$

$$\text{Coefficient: } \frac{\frac{-1}{2}}{1} = \frac{-1}{2}$$

$$\text{Primitive: } -\frac{1}{2} \cdot \arctan(u) + C$$

$$\text{Back-substitute: } -\frac{1}{2} \cdot \arctan\left(-1 - x - 3 \cdot x^2\right) + C$$

$$\text{Domain } g(x): -1 - x - 3 \cdot x^2$$

The inverse kernel permits all real $g(x)$. On each open domain component; constants may differ on disconnected components.

6.

$$\text{Original: } \int \frac{\frac{4}{3}}{\sqrt{9 - \left(2 \cdot \ln\left(-3 - x\right) - 1\right)^2}} dx$$

$$u = 2 \cdot \ln\left(-3 - x\right) - 1$$

$$du = \frac{-2}{-3 - x} \cdot dx$$

$$\text{Scaled: } \frac{\frac{4}{3}}{-3 - x} \cdot dx = \frac{-2}{3} \cdot du$$

$$\text{In } u: \int \frac{\frac{-2}{3}}{\sqrt{9 - u^2}} du$$

$$\text{Coefficient: } \frac{\frac{-2}{3}}{1} = \frac{-2}{3}$$

$$\text{Primitive: } -\frac{2}{3} \cdot \arcsin\left(\frac{u}{3}\right) + C$$

$$\text{Back-substitute: } -\frac{2}{3} \cdot \arcsin\left(\frac{2 \cdot \ln\left(-3 - x\right) - 1}{3}\right) + C$$

$$\text{Domain } g(x): 2 \cdot \ln\left(-3 - x\right) - 1$$

$$\text{Log: } -3 - x > 0$$

$$\text{Inverse kernel: } \left|2 \cdot \ln\left(-3 - x\right) - 1\right| < 3$$

On each open domain component; constants may differ on disconnected components.

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Worked Answer Key

Choose $u = g(x)$, convert the complete differential, integrate the inverse-trigonometric pattern in u , then substitute back and include $+ C$.

7. Original: $\int \frac{2 + 12 \cdot x - 12 \cdot x^2}{\sqrt{1 - (-2 - x - 3 \cdot x^2 + 2 \cdot x^3)^2}} dx$

$u = -2 - x - 3 \cdot x^2 + 2 \cdot x^3$

$du = (-1 - 6 \cdot x + 6 \cdot x^2) dx$

Scaled: $(2 + 12 \cdot x - 12 \cdot x^2) dx = -2 \cdot du$

In u : $\int \frac{-2}{\sqrt{1 - u^2}} du$

Coefficient: $\frac{-2}{1} = -2$

Primitive: $-2 \cdot \arcsin(u) + C$

Back-substitute: $-2 \cdot \arcsin(-2 - x - 3 \cdot x^2 + 2 \cdot x^3) + C$

Domain $g(x)$: $-2 - x - 3 \cdot x^2 + 2 \cdot x^3$

Inverse kernel: $|-2 - x - 3 \cdot x^2 + 2 \cdot x^3| < 1$

On each open domain component; constants may differ on disconnected components.

8. Original: $\int \frac{\frac{-3}{2} \cdot \sec(-2 + 3 \cdot x)^2}{|\tan(-2 + 3 \cdot x) - 1| \sqrt{(\tan(-2 + 3 \cdot x) - 1)^2 - 16}} dx$

$u = \tan(-2 + 3 \cdot x) - 1$

$du = 3 \cdot \sec(-2 + 3 \cdot x)^2 \cdot dx$

Scaled: $\frac{-3}{2} \cdot \sec(-2 + 3 \cdot x)^2 \cdot dx = \frac{-1}{2} \cdot du$

In u : $\int \frac{\frac{-1}{2}}{|u| \sqrt{u^2 - 16}} du$

Coefficient: $\frac{\frac{-1}{2}}{-4} = \frac{1}{8}$

Primitive: $\frac{1}{8} \cdot \operatorname{arccsc}\left(\frac{u}{4}\right) + C$

Back-substitute: $\frac{1}{8} \cdot \operatorname{arccsc}\left(\frac{\tan(-2 + 3 \cdot x) - 1}{4}\right) + C$

Domain $g(x)$: $\tan(-2 + 3 \cdot x) - 1$

Pole: $\cos(-2 + 3 \cdot x) \neq 0$

Inverse kernel: $|\tan(-2 + 3 \cdot x) - 1| > 4$

On each open domain component; constants may differ on disconnected components.