

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Substitution in Definite Integrals

Choose $u = g(x)$, transform both bounds and the differential, then write the equivalent definite integral in u .

1.
$$\int_2^{-3} \left(3 \cdot x + \frac{3}{2} \right) \sin(x^2 + x + 2) dx$$

2.
$$\int_{-3}^{-2} -3 \cdot (-3 \cdot x + 1)^{-2} dx$$

3.
$$\int_1^0 -3 \cdot (2 \cdot x + 2)^1 dx$$

4.
$$\int_{-3}^2 -2 \cdot x \cdot \sqrt{2 \cdot x^2 + 3} dx$$

5.
$$\int_3^1 -18 \cdot x \cdot \cos(-3 \cdot x^2 + 2) dx$$

6.
$$\int_{-3}^{-2} 3 \cdot e^{-3 \cdot x + 2} dx$$

7.
$$\int_3^0 \frac{-1}{2} \cdot \frac{1}{x+3} dx$$

8.
$$\int_{-1}^2 (-3 \cdot x + 1) \cdot (-3 \cdot x^2 + 2 \cdot x - 2)^2 dx$$

Substitution in Definite Integrals

Choose $u = g(x)$, transform both bounds and the differential, then write the equivalent definite integral in u .

9. $\int_3^{-1} \left(6 \cdot x - \frac{3}{2} \right) \cos \left(2 \cdot x^2 - x + 2 \right) dx$

10. $\int_{-3}^{-2} \frac{-2}{3} \cdot \frac{1}{-x+3} dx$

11. $\int_{-3}^0 \left(6 \cdot x + 3 \right) \cdot e^{-3 \cdot x^2 - 3 \cdot x + 3} dx$

12. $\int_{-2}^{-3} \frac{-9}{2} \cdot \left(-3 \cdot x + 3 \right)^{-3} dx$

13. $\int_3^0 \frac{-2}{3} \cdot \sin \left(-x + 2 \right) dx$

14. $\int_2^3 \left(-2 \cdot x - \frac{1}{2} \right) \sqrt{2 \cdot x^2 + x + 1} dx$

15. $\int_3^2 \left(-9 \cdot x - \frac{9}{2} \right) \cdot e^{-3 \cdot x^2 - 3 \cdot x - 1} dx$

16. $\int_{-3}^{-1} \frac{-9}{2} \cdot \frac{1}{-3 \cdot x - 1} dx$

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Substitution in Definite Integrals

Worked Answer Key

Choose $u = g(x)$, transform both bounds and the differential, then write the equivalent definite integral in u .

Transform the endpoints in their original order. Trigonometric arguments use radians.

1. Original: $\int_2^{-3} \left(3 \cdot x + \frac{3}{2}\right) \sin(x^2 + x + 2) dx$

$$u = x^2 + x + 2$$

$$du = (2 \cdot x + 1) dx$$

$$\left(3 \cdot x + \frac{3}{2}\right) dx = \frac{3}{2} du$$

Bounds: $x = 2$ gives $u = 8$; $x = -3$ gives $u = 8$

$$\text{In } u: \int_8^8 \frac{3}{2} \sin(u) du$$

2. Original: $\int_{-3}^{-2} -3 \cdot (-3 \cdot x + 1)^{-2} dx$

$$u = -3 \cdot x + 1$$

$$du = (-3) dx$$

$$(-3) dx = 1 du$$

Bounds: $x = -3$ gives $u = 10$; $x = -2$ gives $u = 7$

$$\text{In } u: \int_{10}^7 u^{-2} du$$

3. Original: $\int_1^0 -3 \cdot (2 \cdot x + 2)^1 dx$

$$u = 2 \cdot x + 2$$

$$du = (2) dx$$

$$(-3) dx = \frac{-3}{2} du$$

Bounds: $x = 1$ gives $u = 4$; $x = 0$ gives $u = 2$

$$\text{In } u: \int_4^2 \frac{-3}{2} \cdot u^1 du$$

4. Original: $\int_{-3}^2 -2 \cdot x \cdot \sqrt{2 \cdot x^2 + 3} dx$

$$u = 2 \cdot x^2 + 3$$

$$du = (4 \cdot x) dx$$

$$(-2 \cdot x) dx = \frac{-1}{2} du$$

Bounds: $x = -3$ gives $u = 21$; $x = 2$ gives $u = 11$

$$\text{In } u: \int_{21}^{11} \frac{-1}{2} \sqrt{u} du$$

Substitution in Definite Integrals

Worked Answer Key

Choose $u = g(x)$, transform both bounds and the differential, then write the equivalent definite integral in u .

Transform the endpoints in their original order. Trigonometric arguments use radians.

5. Original: $\int_3^1 -18 \cdot x \cdot \cos(-3 \cdot x^2 + 2) dx$

$$u = -3 \cdot x^2 + 2$$

$$du = (-6 \cdot x) dx$$

$$(-18 \cdot x) dx = 3 du$$

Bounds: $x = 3$ gives $u = -25$; $x = 1$ gives $u = -1$

$$\text{In } u: \int_{-25}^{-1} 3 \cdot \cos(u) du$$

6. Original: $\int_{-3}^{-2} 3 \cdot e^{-3 \cdot x + 2} dx$

$$u = -3 \cdot x + 2$$

$$du = (-3) dx$$

$$(3) dx = -1 du$$

Bounds: $x = -3$ gives $u = 11$; $x = -2$ gives $u = 8$

$$\text{In } u: \int_{11}^8 -1 \cdot e^u du$$

7. Original: $\int_3^0 \frac{-1}{2} \cdot \frac{1}{x+3} dx$

$$u = x + 3$$

$$du = (1) dx$$

$$\left(\frac{-1}{2}\right) dx = \frac{-1}{2} du$$

Bounds: $x = 3$ gives $u = 6$; $x = 0$ gives $u = 3$

$$\text{In } u: \int_6^3 \frac{-1}{2} \cdot \frac{1}{u} du$$

8. Original: $\int_{-1}^2 (-3 \cdot x + 1) (-3 \cdot x^2 + 2 \cdot x - 2)^2 dx$

$$u = -3 \cdot x^2 + 2 \cdot x - 2$$

$$du = (-6 \cdot x + 2) dx$$

$$(-3 \cdot x + 1) dx = \frac{1}{2} du$$

Bounds: $x = -1$ gives $u = -7$; $x = 2$ gives $u = -10$

$$\text{In } u: \int_{-7}^{-10} \frac{1}{2} \cdot u^2 du$$

Substitution in Definite Integrals

Worked Answer Key

Choose $u = g(x)$, transform both bounds and the differential, then write the equivalent definite integral in u .

Transform the endpoints in their original order. Trigonometric arguments use radians.

9. Original: $\int_3^{-1} \left(6 \cdot x - \frac{3}{2} \right) \cdot \cos \left(2 \cdot x^2 - x + 2 \right) dx$

$$u = 2 \cdot x^2 - x + 2$$

$$du = (4 \cdot x - 1) dx$$

$$\left(6 \cdot x - \frac{3}{2} \right) dx = \frac{3}{2} du$$

Bounds: $x = 3$ gives $u = 17$; $x = -1$ gives $u = 5$

$$\text{In } u: \int_{17}^5 \frac{3}{2} \cdot \cos(u) du$$

10. Original: $\int_{-3}^{-2} \frac{-2}{3} \cdot \frac{1}{-x+3} dx$

$$u = -x + 3$$

$$du = (-1) dx$$

$$\left(\frac{-2}{3} \right) dx = \frac{2}{3} du$$

Bounds: $x = -3$ gives $u = 6$; $x = -2$ gives $u = 5$

$$\text{In } u: \int_6^5 \frac{2}{3} \cdot \frac{1}{u} du$$

11. Original: $\int_{-3}^0 (6 \cdot x + 3) e^{-3 \cdot x^2 - 3 \cdot x + 3} dx$

$$u = -3 \cdot x^2 - 3 \cdot x + 3$$

$$du = (-6 \cdot x - 3) dx$$

$$(6 \cdot x + 3) dx = -1 du$$

Bounds: $x = -3$ gives $u = -15$; $x = 0$ gives $u = 3$

$$\text{In } u: \int_{-15}^3 -1 \cdot e^u du$$

12. Original: $\int_{-2}^{-3} \frac{-9}{2} \cdot (-3 \cdot x + 3)^{-3} dx$

$$u = -3 \cdot x + 3$$

$$du = (-3) dx$$

$$\left(\frac{-9}{2} \right) dx = \frac{3}{2} du$$

Bounds: $x = -2$ gives $u = 9$; $x = -3$ gives $u = 12$

$$\text{In } u: \int_9^{12} \frac{3}{2} \cdot u^{-3} du$$

Substitution in Definite Integrals

Worked Answer Key

Choose $u = g(x)$, transform both bounds and the differential, then write the equivalent definite integral in u .

Transform the endpoints in their original order. Trigonometric arguments use radians.

13. Original: $\int_3^0 \frac{-2}{3} \cdot \sin(-x + 2) dx$

$$u = -x + 2$$

$$du = (-1) dx$$

$$\left(\frac{-2}{3}\right) dx = \frac{2}{3} du$$

Bounds: $x = 3$ gives $u = -1$; $x = 0$ gives $u = 2$

$$\text{In } u: \int_{-1}^2 \frac{2}{3} \cdot \sin(u) du$$

14. Original: $\int_2^3 \left(-2x - \frac{1}{2}\right) \sqrt{2x^2 + x + 1} dx$

$$u = 2x^2 + x + 1$$

$$du = (4x + 1) dx$$

$$\left(-2x - \frac{1}{2}\right) dx = \frac{-1}{2} du$$

Bounds: $x = 2$ gives $u = 11$; $x = 3$ gives $u = 22$

$$\text{In } u: \int_{11}^{22} \frac{-1}{2} \sqrt{u} du$$

15. Original: $\int_3^2 \left(-9x - \frac{9}{2}\right) e^{-3x^2 - 3x - 1} dx$

$$u = -3x^2 - 3x - 1$$

$$du = (-6x - 3) dx$$

$$\left(-9x - \frac{9}{2}\right) dx = \frac{3}{2} du$$

Bounds: $x = 3$ gives $u = -37$; $x = 2$ gives $u = -19$

$$\text{In } u: \int_{-37}^{-19} \frac{3}{2} e^u du$$

16. Original: $\int_{-3}^{-1} \frac{-9}{2} \cdot \frac{1}{3x - 1} dx$

$$u = 3x - 1$$

$$du = (3) dx$$

$$\left(\frac{-9}{2}\right) dx = \frac{3}{2} du$$

Bounds: $x = -3$ gives $u = -10$; $x = -1$ gives $u = -4$

$$\text{In } u: \int_{-10}^{-4} \frac{3}{2} \cdot \frac{1}{u} du$$