

Series Convergence Tests and Error Bounds

Choose and apply a valid convergence test, classify absolute, conditional, or divergent behavior, and show the selected remainder bound when requested.

1. Use the ratio test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{n!}{8^n}$ as absolute, conditional, or divergent. If it converges, bound $|R_{13}|$.

2. Use the direct comparison test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+5)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{14}|$.

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Choose and apply a valid convergence test, classify absolute, conditional, or divergent behavior, and show the selected remainder bound when requested.

3. Use the limit comparison test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{1}{(n+2)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{11}|$.

4. Use the alternating test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+7)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{14}|$.

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5. Use the root test as far as it applies to classify $\sum_{n=1}^{\infty} (1+\frac{1}{7})^n$ as absolute, conditional, or divergent. If it converges, bound $|R_9|$.

6. Use the direct comparison test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{1}{(n+7)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{14}|$.

Series Convergence Tests and Error Bounds

Worked Answer Key

Choose and apply a valid convergence test, classify absolute, conditional, or divergent behavior, and show the selected remainder bound when requested.

1. Use the ratio test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{n!}{8^n}$ as absolute, conditional, or divergent. If it converges, bound $|R_{13}|$.

Answer: The series diverges. No convergent remainder exists.

1. The ratio grows without bound, hence exceeds 1.

2. Therefore the series diverges.

3. A remainder bound is not defined for a divergent series.

2. Use the direct comparison test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+5)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{14}|$.

Answer: The series converges conditionally. $|R_{14}| \leq 1/20$.

1. For $n \geq 1$, $1/(n+5) \geq 1/[(6)n]$, so the absolute series diverges by direct comparison; the alternating test gives convergence.

2. Therefore the series converges conditionally.

3. The alternating-series remainder theorem gives $|R_{14}| \leq b_{15} = 1/20$.

Series Convergence Tests and Error Bounds

Worked Answer Key

Choose and apply a valid convergence test, classify absolute, conditional, or divergent behavior, and show the selected remainder bound when requested.

3. Use the limit comparison test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{1}{(n+2)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{11}|$.

Answer: The series diverges. No convergent remainder exists.

- The limit comparison with $1/n$ is 1.
- Therefore the series diverges.
- A remainder bound is not defined for a divergent series.

4. Use the alternating test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+7)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{14}|$.

Answer: The series converges conditionally. $|R_{14}| \leq 1/22$.

- The magnitudes decrease to 0, so the alternating series converges; the absolute shifted harmonic series diverges.
- Therefore the series converges conditionally.
- The alternating-series remainder theorem gives $|R_{14}| \leq b_{15} = 1/22$.

Series Convergence Tests and Error Bounds

Worked Answer Key

Choose and apply a valid convergence test, classify absolute, conditional, or divergent behavior, and show the selected remainder bound when requested.

5. Use the root test as far as it applies to classify $\sum_{n=1}^{\infty} (1+\frac{1}{7})^n$ as absolute, conditional, or divergent. If it converges, bound $|R_9|$.

Answer: The series diverges. No convergent remainder exists.

- The root limit is $1.142857 > 1$.
- Therefore the series diverges.
- A remainder bound is not defined for a divergent series.

6. Use the direct comparison test as far as it applies to classify $\sum_{n=1}^{\infty} \frac{1}{(n+7)}$ as absolute, conditional, or divergent. If it converges, bound $|R_{14}|$.

Answer: The series diverges. No convergent remainder exists.

- $\frac{1}{(n+7)} \geq \frac{1}{(8n)}$, a divergent harmonic multiple.
- Therefore the series diverges.
- A remainder bound is not defined for a divergent series.