

Sequence Limits and Partial Sums

Distinguish sequence limits from series partial sums. Show divergence explicitly and justify monotone/bounded conclusions.

1. Let $a_1 = -5$ and $a_{n+1} = (a_n + 0)/2$. Prove the sequence is monotone and bounded, then state its limit.

2. Let $a_1 = 21$ and $a_{n+1} = (a_n + 7)/2$. Determine $\lim_{n \rightarrow \infty} a_n$.

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3. For $a_n = 1 + 4(-1)^n$, determine $\lim_{(n \rightarrow \infty)} a_n$. State whether the sequence converges or diverges.

4. For the recursive sequence $a_1=6$, $a_{(n+1)}=(-1/4)a_n$, find the finite partial sum $S_8=a_1+...+a_8$. Do not replace it with the infinite-series sum.

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5. For the explicit sequence $a_n=8(1/2)^{(n-1)}$, find the finite partial sum $S_7=a_1+...+a_7$. Do not replace it with the infinite-series sum.

6. For $a_n=3 + 8/n^3$, decide whether the sequence is increasing/decreasing and bounded. Then state its limit.

Sequence Limits and Partial Sums

Worked Answer Key

Distinguish sequence limits from series partial sums. Show divergence explicitly and justify monotone/bounded conclusions.

1. Let $a_1 = -5$ and $a_{n+1} = (a_n + 0)/2$. Prove the sequence is monotone and bounded, then state its limit.

Answer: Increasing and bounded above; limit 0.

1. $a_{n+1} - 0 = (a_n - 0)/2$ preserves the side of 0.

2. The distance to 0 is halved each step, so terms move monotonically toward 0.

3. Monotone convergence gives $\lim a_n = 0$.

2. Let $a_1 = 21$ and $a_{n+1} = (a_n + 7)/2$. Determine $\lim_{n \rightarrow \infty} a_n$.

Answer: The limit is 7.

1. Subtract 7: $a_{n+1} - 7 = (a_n - 7)/2$.

2. Thus $a_n - 7 = (14)(1/2)^{(n-1)} \rightarrow 0$.

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3. For $a_n = 1 + 4(-1)^n$, determine $\lim_{n \rightarrow \infty} a_n$. State whether the sequence converges or diverges.

Answer: The limit does not exist; the sequence diverges.

- Even and odd subsequences are constant at 5 and -3.
- Because the two subsequential limits differ, the sequence limit does not exist.

4. For the recursive sequence $a_1=6$, $a_{n+1}=(-1/4)a_n$, find the finite partial sum $S_8=a_1+\dots+a_8$. Do not replace it with the infinite-series sum.

Answer: $S_8=6(1-(-1/4)^8)/(1-(-1/4))$, approximately 4.799927.

- This is a finite geometric sum with first term 6 and ratio -1/4.
- $S_N=a_1(1-r^N)/(1-r)$, so $S_8=6(1-(-1/4)^8)/(1-(-1/4))$.

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Worked Answer Key

Distinguish sequence limits from series partial sums. Show divergence explicitly and justify monotone/bounded conclusions.

5. For the explicit sequence $a_n=8(1/2)^{(n-1)}$, find the finite partial sum $S_7=a_1+...+a_7$. Do not replace it with the infinite-series sum.

Answer: $S_7=8(1-(1/2)^7)/(1-1/2)$, approximately 15.875.

- 1. This is a finite geometric sum with first term 8 and ratio 1/2.
- 2. $S_N=a_1(1-r^N)/(1-r)$, so $S_7=8(1-(1/2)^7)/(1-1/2)$.

6. For $a_n=3 + 8/n^3$, decide whether the sequence is increasing/decreasing and bounded. Then state its limit.

Answer: Decreasing, bounded below by 3, and convergent to 3.

- 1. $1/n^3$ decreases to 0.
- 2. $+8/n^3$ therefore decreases toward 0.
- 3. The sequence is bounded below by 3, so its limit is 3.