

Solving Separable Equations

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

1.

$$\frac{dy}{dx} = \frac{-1 \cdot \cos(3 \cdot x + 1)}{y^2}$$

Implicit solution and equilibria: _____

2.

$$\frac{dy}{dx} = -9 \cdot x \cdot y$$

Implicit solution and equilibria: _____

3.

$$\frac{dy}{dx} = -7 \cdot e^{-2 \cdot x} \cdot y^2$$

Implicit solution and equilibria: _____

4.

$$\frac{dy}{dx} = \frac{-5 \cdot x^3}{y}$$

Implicit solution and equilibria: _____

5.

$$\frac{dy}{dx} = -3 \cdot \sin(x - 3) \cdot y^3$$

Implicit solution and equilibria: _____

6.

$$\frac{dy}{dx} = \frac{-1}{2} \cdot x \cdot y^2$$

Implicit solution and equilibria: _____

7.

$$\frac{dy}{dx} = \frac{3}{2} \cdot x \cdot e^{3 \cdot y}$$

Implicit solution and equilibria: _____

8.

$$\frac{dy}{dx} = \frac{5 \cdot x^3}{y^2}$$

Implicit solution and equilibria: _____

9.

$$\frac{dy}{dx} = \frac{-2}{3} \cdot \sec(-1 \cdot x)^2 \cdot y^3$$

Implicit solution and equilibria: _____

10.

$$\frac{dy}{dx} = 5 \cdot x^3 \cdot y^3$$

Implicit solution and equilibria: _____

11.

$$\frac{dy}{dx} = x^4 \cdot \sin(3 \cdot y)^2$$

Implicit solution and equilibria: _____

12.

$$\frac{dy}{dx} = \frac{-2 \cdot x}{y^3}$$

Implicit solution and equilibria: _____

Solving Separable Equations

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

13.

$$\frac{dy}{dx} = \frac{3}{2} \cdot \csc(-1 \cdot x)^2 \cdot y$$

Implicit solution and equilibria: _____

14.

$$\frac{dy}{dx} = 6 \cdot x^0 \cdot y$$

Implicit solution and equilibria: _____

15.

$$\frac{dy}{dx} = \frac{\frac{1}{2} \cdot e^x}{y^2}$$

Implicit solution and equilibria: _____

16.

$$\frac{dy}{dx} = \frac{3 \cdot x}{y}$$

Implicit solution and equilibria: _____

17.

$$\frac{dy}{dx} = \frac{\frac{2}{3} \cdot \sec(-3 \cdot x - 1) \cdot \tan(-3 \cdot x - 1)}{y^3}$$

Implicit solution and equilibria: _____

18.

$$\frac{dy}{dx} = 7 \cdot x^3 \cdot y^2$$

Implicit solution and equilibria: _____

19.

$$\frac{dy}{dx} = -4 \cdot x^0 \cdot e^{-1 \cdot y}$$

Implicit solution and equilibria: _____

20.

$$\frac{dy}{dx} = \frac{3 \cdot x}{y^2}$$

Implicit solution and equilibria: _____

21.

$$\frac{dy}{dx} = \csc(-x + 1) \cdot \cot(-x + 1) \cdot y^2$$

Implicit solution and equilibria: _____

22.

$$\frac{dy}{dx} = 4 \cdot x^2 \cdot y^3$$

Implicit solution and equilibria: _____

23.

$$\frac{dy}{dx} = -3 \cdot x^3 \cdot \cos(2 \cdot y)^2$$

Implicit solution and equilibria: _____

24.

$$\frac{dy}{dx} = \frac{-4 \cdot x^0}{y^3}$$

Implicit solution and equilibria: _____

Solving Separable Equations

Worked Answer Key

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

1. Separate and solve

$$\frac{dy}{dx} = \frac{-1 \cdot \cos(3 \cdot x + 1)}{y^2}$$

$$\text{integral } y^2 dy = \text{integral } -1 \cdot \cos(3 \cdot x + 1) dx$$

Separate the y power on an interval that avoids each original x pole. Integrate the certified cosine pattern with its affine chain adjustment. Use one + C and restore any equilibrium solution.

$$\text{Implicit: } \frac{1}{3} \cdot y^3 = -\frac{1}{3} \cdot \sin(3 \cdot x + 1) + C$$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

$$\text{Check: } y^2 * \frac{dy}{dx} = -1 \cdot \cos(3 \cdot x + 1)$$

Differentiate the implicit equality, then substitute dy/dx = X(x)H(y); the left side simplifies to X(x), matching the right side.

2. Separate and solve

$$\frac{dy}{dx} = -9 \cdot x \cdot y$$

$$\text{integral } \frac{1}{y} dy = \text{integral } -9 \cdot x dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \ln(|y|) = -\frac{9}{2} \cdot x^2 + C$$

Equilibria: y = 0.

Original domain: all real x and y. Nonconstant branch: y is not 0.

$$\text{Check: } \frac{1}{y} * \frac{dy}{dx} = -9 \cdot x$$

Differentiate the implicit equality, then substitute dy/dx = X(x)H(y); the left side simplifies to X(x), matching the right side.

3. Separate and solve

$$\frac{dy}{dx} = -7 \cdot e^{-2 \cdot x} \cdot y^2$$

$$\text{integral } y^{-2} dy = \text{integral } -7 \cdot e^{-2 \cdot x} dx$$

Separate the y power from the exponential x factor. Integrate e^{a(x)} with the exact 1/a chain adjustment. Use one + C and restore any y = 0 equilibrium.

$$\text{Implicit: } -1 \cdot y^{-1} = \frac{7}{2} \cdot e^{-2 \cdot x} + C$$

Equilibria: y = 0.

Original domain: all real x and y. Nonconstant branch: y is not 0.

$$\text{Check: } y^{-2} * \frac{dy}{dx} = -7 \cdot e^{-2 \cdot x}$$

Differentiate the implicit equality, then substitute dy/dx = X(x)H(y); the left side simplifies to X(x), matching the right side.

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Worked Answer Key

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
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4. Separate and solve

$$\frac{dy}{dx} = \frac{-5 \cdot x^3}{y}$$

$$\text{integral } y \, dy = \text{integral } -5 \cdot x^3 \, dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \frac{1}{2} \cdot y^2 = -\frac{5}{4} \cdot x^4 + C$$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

$$\text{Check: } y \cdot \frac{dy}{dx} = -5 \cdot x^3$$

Differentiate the implicit equality, then substitute dy/dx = X(x)H(y); the left side simplifies to X(x), matching the right side.

5. Separate and solve

$$\frac{dy}{dx} = -3 \cdot \sin(x - 3) \cdot y^3$$

$$\text{integral } y^{-3} \, dy = \text{integral } -3 \cdot \sin(x - 3) \, dx$$

Separate the y power on an interval that avoids each original x pole. Integrate the certified sine pattern with its affine chain adjustment. Use one + C and restore any equilibrium solution.

$$\text{Implicit: } \frac{-1}{2} \cdot y^{-2} = 3 \cdot \cos(x - 3) + C$$

Equilibria: y = 0.

Original domain: all real x and y. Nonconstant branch: y is not 0.

$$\text{Check: } y^{-3} \cdot \frac{dy}{dx} = -3 \cdot \sin(x - 3)$$

Differentiate the implicit equality, then substitute dy/dx = X(x)H(y); the left side simplifies to X(x), matching the right side.

6. Separate and solve

$$\frac{dy}{dx} = \frac{-1}{2} \cdot x \cdot y^2$$

$$\text{integral } y^{-2} \, dy = \text{integral } \frac{-1}{2} \cdot x \, dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } -1 \cdot y^{-1} = -\frac{1}{4} \cdot x^2 + C$$

Equilibria: y = 0.

Original domain: all real x and y. Nonconstant branch: y is not 0.

$$\text{Check: } y^{-2} \cdot \frac{dy}{dx} = \frac{-1}{2} \cdot x$$

Differentiate the implicit equality, then substitute dy/dx = X(x)H(y); the left side simplifies to X(x), matching the right side.

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Worked Answer Key

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

7. Separate and solve

$$\frac{dy}{dx} = \frac{3}{2} \cdot x \cdot e^{3 \cdot y}$$

$$\text{integral } e^{-1 \cdot 3 \cdot y} dy = \text{integral } \frac{3}{2} \cdot x dx$$

Multiply by $e^{(-by)}$ to separate y from x. Integrate both sides using the exponential chain rule and the x power rule. Combine constants once as + C; the exponential factor has no equilibrium zeros.

$$\text{Implicit: } \frac{-1}{3} \cdot e^{-1 \cdot 3 \cdot y} = \frac{3}{4} \cdot x^2 + C$$

Equilibria: none.

Original domain: all real x and y. Nonconstant branch: no additional restrictions.

$$\text{Check: } e^{-1 \cdot 3 \cdot y} \cdot \frac{dy}{dx} = \frac{3}{2} \cdot x$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

8. Separate and solve

$$\frac{dy}{dx} = \frac{5 \cdot x^3}{y^2}$$

$$\text{integral } y^2 dy = \text{integral } 5 \cdot x^3 dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \frac{1}{3} \cdot y^3 = \frac{5}{4} \cdot x^4 + C$$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

$$\text{Check: } y^2 \cdot \frac{dy}{dx} = 5 \cdot x^3$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

9. Separate and solve

$$\frac{dy}{dx} = \frac{-2}{3} \cdot \sec(-1 \cdot x)^2 \cdot y^3$$

$$\text{integral } y^{-3} dy = \text{integral } \frac{-2}{3} \cdot \sec(-1 \cdot x)^2 dx$$

Separate the y power on an interval that avoids each original x pole. Integrate the certified secant-squared pattern with its affine chain adjustment. Use one + C and restore any equilibrium solution.

$$\text{Implicit: } \frac{-1}{2} \cdot y^{-2} = \frac{2}{3} \cdot \tan(-1 \cdot x) + C$$

Equilibria: $y = 0$.

Original domain: $\cos(-1x+0)$ is not 0. Nonconstant branch: $\cos(-1x+0)$ is not 0; y is not 0.

$$\text{Check: } y^{-3} \cdot \frac{dy}{dx} = \frac{-2}{3} \cdot \sec(-1 \cdot x)^2$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

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Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

10. Separate and solve

$$\frac{dy}{dx} = 5 \cdot x^3 \cdot y^3$$

$$\text{integral } y^{-3} dy = \text{integral } 5 \cdot x^3 dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \frac{-1}{2} \cdot y^{-2} = \frac{5}{4} \cdot x^4 + C$$

Equilibria: $y = 0$.

Original domain: all real x and y. Nonconstant branch: y is not 0.

$$\text{Check: } y^{-3} * \frac{dy}{dx} = 5 \cdot x^3$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

11. Separate and solve

$$\frac{dy}{dx} = x^4 \cdot \sin(3 \cdot y)^2$$

$$\text{integral } \csc(3 \cdot y)^2 dy = \text{integral } x^4 dx$$

Divide by $\sin(3y)^2$ for the nonconstant branches. Integrate the resulting $\csc^2$ factor and the x power. Use one + C, then restore the complete integer-indexed equilibrium family.

$$\text{Implicit: } \frac{-1}{3} \cdot \cot(3 \cdot y) = \frac{1}{5} \cdot x^5 + C$$

Equilibria: $y = k\pi/3$, k any integer.

Original domain: all real x and y. Nonconstant branch: $\sin(3y)$ is not 0 on a nonconstant branch.

$$\text{Check: } \csc(3 \cdot y)^2 * \frac{dy}{dx} = x^4$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

12. Separate and solve

$$\frac{dy}{dx} = \frac{-2 \cdot x}{y^3}$$

$$\text{integral } y^3 dy = \text{integral } -2 \cdot x dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \frac{1}{4} \cdot y^4 = -\frac{1}{2} \cdot x^2 + C$$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

$$\text{Check: } y^3 * \frac{dy}{dx} = -2 \cdot x$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

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Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
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13. Separate and solve

$$\frac{dy}{dx} = \frac{3}{2} \cdot \csc(-1 \cdot x)^2 \cdot y$$

$$\text{integral } \frac{1}{y} dy = \text{integral } \frac{3}{2} \cdot \csc(-1 \cdot x)^2 dx$$

Separate the y power on an interval that avoids each original x pole. Integrate the certified cosecant-squared pattern with its affine chain adjustment. Use one + C and restore any equilibrium solution.

Implicit: $\ln(|y|) = \frac{3}{2} \cdot \cot(-1 \cdot x) + C$

Equilibria: $y = 0$.

Original domain: $\sin(-1x+0)$ is not 0. Nonconstant branch: $\sin(-1x+0)$ is not 0; y is not 0.

Check: $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{2} \cdot \csc(-1 \cdot x)^2$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

14. Separate and solve

$$\frac{dy}{dx} = 6 \cdot x^0 \cdot y$$

$$\text{integral } \frac{1}{y} dy = \text{integral } 6 dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

Implicit: $\ln(|y|) = 6 \cdot x + C$

Equilibria: $y = 0$.

Original domain: all real x and y. Nonconstant branch: y is not 0.

Check: $\frac{1}{y} \cdot \frac{dy}{dx} = 6$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

Solving Separable Equations

Worked Answer Key

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

15. Separate and solve

$$\frac{dy}{dx} = \frac{\frac{1}{2} \cdot e^x}{y^2}$$

$$\text{integral } y^2 dy = \text{integral } \frac{1}{2} \cdot e^x dx$$

Separate the y power from the exponential x factor. Integrate e^x with the exact $1/a$ chain adjustment. Use one + C and restore any $y = 0$ equilibrium.

Implicit: $\frac{1}{3} \cdot y^3 = \frac{1}{2} \cdot e^x + C$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

Check: $y^2 \cdot \frac{dy}{dx} = \frac{1}{2} \cdot e^x$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

16. Separate and solve

$$\frac{dy}{dx} = \frac{3 \cdot x}{y}$$

$$\text{integral } y dy = \text{integral } 3 \cdot x dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

Implicit: $\frac{1}{2} \cdot y^2 = \frac{3}{2} \cdot x^2 + C$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

Check: $y \cdot \frac{dy}{dx} = 3 \cdot x$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

Solving Separable Equations

Worked Answer Key

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

17. Separate and solve

$$\frac{dy}{dx} = \frac{\frac{2}{3} \cdot \sec(-3x-1) \cdot \tan(-3x-1)}{y^3}$$

$$\text{integral } y^3 dy = \text{integral } \frac{2}{3} \cdot \sec(-3x-1) \cdot \tan(-3x-1) dx$$

Separate the y power on an interval that avoids each original x pole. Integrate the certified secant-tangent pattern with its affine chain adjustment. Use one + C and restore any equilibrium solution.

Implicit: $\frac{1}{4} \cdot y^4 = -\frac{2}{9} \cdot \sec(-3x-1) + C$

Equilibria: none.

Original domain: $\cos(-3x-1)$ is not 0; y is not 0. Nonconstant branch: $\cos(-3x-1)$ is not 0; y is not 0.

Check: $y^3 \cdot \frac{dy}{dx} = \frac{2}{3} \cdot \sec(-3x-1) \cdot \tan(-3x-1)$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

18. Separate and solve

$$\frac{dy}{dx} = 7 \cdot x^3 \cdot y^2$$

$$\text{integral } y^{-2} dy = \text{integral } 7 \cdot x^3 dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

Implicit: $-1 \cdot y^{-1} = \frac{7}{4} \cdot x^4 + C$

Equilibria: $y = 0$.

Original domain: all real x and y. Nonconstant branch: y is not 0.

Check: $y^{-2} \cdot \frac{dy}{dx} = 7 \cdot x^3$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

Solving Separable Equations

Worked Answer Key

Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
Interpret nonconstant branches on connected intervals in the original domain and where division by the y-factor is valid.

19. Separate and solve

$$\frac{dy}{dx} = -4 \cdot x^0 \cdot e^{-1 \cdot y}$$

$$\text{integral } e^{-1 \cdot -1 \cdot y} dy = \text{integral } -4 \cdot dx$$

Multiply by $e^{(-by)}$ to separate y from x. Integrate both sides using the exponential chain rule and the x power rule. Combine constants once as + C; the exponential factor has no equilibrium zeros.

$$\text{Implicit: } e^{-1 \cdot -1 \cdot y} = -4 \cdot x + C$$

Equilibria: none.

Original domain: all real x and y. Nonconstant branch: no additional restrictions.

$$\text{Check: } e^{-1 \cdot -1 \cdot y} \star \frac{dy}{dx} = -4$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

20. Separate and solve

$$\frac{dy}{dx} = \frac{3 \cdot x}{y^2}$$

$$\text{integral } y^2 dy = \text{integral } 3 \cdot x dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \frac{1}{3} \cdot y^3 = \frac{3}{2} \cdot x^2 + C$$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

$$\text{Check: } y^2 \star \frac{dy}{dx} = 3 \cdot x$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

21. Separate and solve

$$\frac{dy}{dx} = \csc(-x+1) \cdot \cot(-x+1) \cdot y^2$$

$$\text{integral } y^{-2} dy = \text{integral } \csc(-x+1) \cdot \cot(-x+1) dx$$

Separate the y power on an interval that avoids each original x pole. Integrate the certified cosecant-cotangent pattern with its affine chain adjustment. Use one + C and restore any equilibrium solution.

$$\text{Implicit: } -1 \cdot y^{-1} = \csc(-x+1) + C$$

Equilibria: $y = 0$.

Original domain: $\sin(-1x+1)$ is not 0. Nonconstant branch: $\sin(-1x+1)$ is not 0; y is not 0.

$$\text{Check: } y^{-2} \star \frac{dy}{dx} = \csc(-x+1) \cdot \cot(-x+1)$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

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Separate variables, integrate both sides, and give an implicit general solution with one C and every equilibrium solution.
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22. Separate and solve

$$\frac{dy}{dx} = 4 \cdot x^2 \cdot y^3$$

$$\text{integral } y^{-3} dy = \text{integral } 4 \cdot x^2 dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \frac{-1}{2} \cdot y^{-2} = \frac{4}{3} \cdot x^3 + C$$

Equilibria: $y = 0$.

Original domain: all real x and y. Nonconstant branch: y is not 0.

$$\text{Check: } y^{-3} * \frac{dy}{dx} = 4 \cdot x^2$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

23. Separate and solve

$$\frac{dy}{dx} = -3 \cdot x^3 \cdot \cos(2 \cdot y)^2$$

$$\text{integral } \sec(2 \cdot y)^2 dy = \text{integral } -3 \cdot x^3 dx$$

Divide by $\cos(2y)^2$ for the nonconstant branches. Integrate the resulting $\sec^2$ factor and the x power. Use one + C, then restore the complete integer-indexed equilibrium family.

$$\text{Implicit: } \frac{1}{2} \cdot \tan(2 \cdot y) = -\frac{3}{4} \cdot x^4 + C$$

Equilibria: $y = \pi/4 + k \cdot \pi/2$, k any integer.

Original domain: all real x and y. Nonconstant branch: $\cos(2y)$ is not 0 on a nonconstant branch.

$$\text{Check: } \sec(2 \cdot y)^2 * \frac{dy}{dx} = -3 \cdot x^3$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.

24. Separate and solve

$$\frac{dy}{dx} = \frac{-4 \cdot x^0}{y^3}$$

$$\text{integral } y^3 dy = \text{integral } -4 dx$$

Divide by the y-dependent factor and separate dy from dx. Integrate both sides with exact power rules. Combine constants once as + C and restore every equilibrium solution.

$$\text{Implicit: } \frac{1}{4} \cdot y^4 = -4 \cdot x + C$$

Equilibria: none.

Original domain: y is not 0. Nonconstant branch: y is not 0.

$$\text{Check: } y^3 * \frac{dy}{dx} = -4$$

Differentiate the implicit equality, then substitute $dy/dx = X(x)H(y)$; the left side simplifies to $X(x)$, matching the right side.