

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Related Rates

Write the constraint, differentiate with respect to time, then substitute the stated instant.

1. cone tank

Water in an inverted cone with its vertex at the bottom is 9 ft high and 3 ft in radius. The water depth is 4 ft and is increasing at 2 ft/min. How fast is the water volume changing?

Set-up and work:



$$R/H = 3/9$$
$$h = 4$$

2. sliding ladder

A 17 ft ladder rests against a wall. When its foot is 8 ft from the wall and moving away from the wall at 2 ft/s, how fast is the top moving?

Set-up and work:



$$x = 8$$
$$y = 15$$
$$\text{ladder} = 17$$

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3. sphere volume

A spherical balloon has radius 2 m and its radius is shrinking at 2 m/min. How fast is its volume changing then?

Set-up and work:



$r = 2 \text{ m}$

4. circle area

A circular ripple has radius 5 cm and its radius is expanding at 2 cm/s. How fast is its area changing at that instant?

Set-up and work:



$r = 5 \text{ cm}$

Related Rates

Worked Answer Key

Write the constraint, differentiate with respect to time, then substitute the stated instant.

1. cone tank

Water in an inverted cone with its vertex at the bottom is 9 ft high and 3 ft in radius. The water depth is 4 ft and is increasing at 2 ft/min. How fast is the water volume changing?

Similarity:

$$r/h = \frac{1}{3}$$

Constraint:

$$V = \frac{1}{27} \cdot \pi \cdot h^3$$

Differentiate first:

$$dV/dt = \frac{1}{9} \cdot \pi \cdot h^2 \cdot dh/dt$$

Substitute:

$$dV/dt = \frac{32}{9} \cdot \pi$$

Answer: $32/9 \pi \text{ ft}^3/\text{min}$.

Related Rates

Worked Answer Key

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2. sliding ladder

A 17 ft ladder rests against a wall. When its foot is 8 ft from the wall and moving away from the wall at 2 ft/s, how fast is the top moving?

Constraint:

$$x^2 + y^2 = 289$$

Differentiate first:

$$2 \cdot x \cdot dx/dt + 2 \cdot y \cdot dy/dt = 0$$

Substitute:

$$dy/dt = \frac{-16}{15}$$

Answer: -16/15 ft/s (negative means decreasing).

Related Rates

Worked Answer Key

Write the constraint, differentiate with respect to time, then substitute the stated instant.

3. sphere volume

A spherical balloon has radius 2 m and its radius is shrinking at 2 m/min. How fast is its volume changing then?

Constraint:

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

Differentiate first:

$$dV/dt = 4 \cdot \pi \cdot r^2 \cdot dr/dt$$

Substitute:

$$dV/dt = -32 \cdot \pi$$

Answer: $-32 \pi \text{ m}^3/\text{min}$ (negative means decreasing).

Related Rates

Worked Answer Key

Write the constraint, differentiate with respect to time, then substitute the stated instant.

4. circle area

A circular ripple has radius 5 cm and its radius is expanding at 2 cm/s. How fast is its area changing at that instant?

Constraint:

$$A = \pi \cdot r^2$$

Differentiate first:

$$dA/dt = 2 \cdot \pi \cdot r \cdot dr/dt$$

Substitute:

$$dA/dt = 20 \cdot \pi$$

Answer: 20 pi cm²/s.