

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Power Series Radius and Interval of Convergence

Find the radius first, then test the left and right endpoints separately. Recheck endpoint behavior after termwise differentiation or integration.

1. Use $\sum_{n=1}^{\infty} \frac{2}{(6^n n^3)}(x-0)^n$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-6$ and $x=6$.

2. Use $\sum_{n=1}^{\infty} \frac{4}{(3^n n^4)}(x+3)^n$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-6$ and $x=0$.

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Power Series Radius and Interval of Convergence

Find the radius first, then test the left and right endpoints separately. Recheck endpoint behavior after termwise differentiation or integration.

3. Integrate $\sum_{n=1}^{\infty} \frac{5}{2^n n^1} (x+3)^n$ term by term with constant $C=0$ to obtain $\sum_{n=1}^{\infty} \frac{5(x+3)^{n+1}}{[2^n n^1(n+1)]}$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-5$ and $x=-1$.

4. Use $\sum_{n=1}^{\infty} \frac{4}{6^n} (x-1)^n$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-5$ and $x=7$.

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Power Series Radius and Interval of Convergence

Find the radius first, then test the left and right endpoints separately. Recheck endpoint behavior after termwise differentiation or integration.

5. Differentiate $\sum_{n=1}^{\infty} \frac{5}{7^n n^2} (x-2)^n$ term by term to obtain $\sum_{n=1}^{\infty} \frac{5n(x-2)^{n-1}}{7^n n^2}$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-5$ and $x=9$.

6. Integrate $\sum_{n=1}^{\infty} \frac{4n^{1/7}}{7^n} (x-0)^n$ term by term with constant $C=0$ to obtain $\sum_{n=1}^{\infty} \frac{4(x-0)^{n+1}}{7^n n^{1/7} (n+1)}$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-7$ and $x=7$.

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Power Series Radius and Interval of Convergence

Worked Answer Key

Find the radius first, then test the left and right endpoints separately. Recheck endpoint behavior after termwise differentiation or integration.

1. Use $\sum_{n=1}^{\infty} \frac{2}{(6^n n^3)}(x-0)^n$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-6$ and $x=6$.

Answer: Resulting series: $\sum_{n=1}^{\infty} \frac{2}{(6^n n^3)}(x-0)^n$. Radius $R=6$; interval $[-6, 6]$.

1. The ratio/root calculation gives $|x-0|<6$, so the radius is 6.
2. At $x=-6$, substitution gives $\sum 2(-1)^n/n^3$. The magnitudes are constant multiples of $1/n^3$. The signs alternate, so the left endpoint converges absolutely.
3. At $x=6$, substitution gives $\sum 2/n^3$. The magnitudes are constant multiples of $1/n^3$. The terms are positive, so the right endpoint converges.
4. Therefore the interval is $[-6, 6]$.

2. Use $\sum_{n=1}^{\infty} \frac{4}{(3^n n^1)}(x+3)^n$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-6$ and $x=0$.

Answer: Resulting series: $\sum_{n=1}^{\infty} \frac{4}{(3^n n^1)}(x+3)^n$. Radius $R=3$; interval $[-6, 0)$.

1. The ratio/root calculation gives $|x+3|<3$, so the radius is 3.
2. At $x=-6$, substitution gives $\sum 4(-1)^n/n^1$. The magnitudes are constant multiples of $1/n^1$. The signs alternate, so the left endpoint converges conditionally.
3. At $x=0$, substitution gives $\sum 4/n^1$. The magnitudes are constant multiples of $1/n^1$. The terms are positive, so the right endpoint diverges as a harmonic-type series.
4. Therefore the interval is $[-6, 0)$.

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Power Series Radius and Interval of Convergence

Worked Answer Key

Find the radius first, then test the left and right endpoints separately. Recheck endpoint behavior after termwise differentiation or integration.

3. Integrate $\sum_{n=1}^{\infty} \frac{5(2^n n^{n+1})(x+3)^n}{n^{n+1}}$ term by term with constant $C=0$ to obtain $\sum_{n=1}^{\infty} \frac{5(x+3)^{n+1}}{2^n n^{n+1}}$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-5$ and $x=-1$.

Answer: Resulting series: $\sum_{n=1}^{\infty} \frac{5(x+3)^{n+1}}{2^n n^{n+1}}$. Radius $R=2$; interval $[-5, -1]$.

1. The ratio/root calculation gives $|x+3| < 2$, so the radius is 2.
2. At $x=-5$, substitution gives $\sum 10(-1)^{n+1}/[n^{n+1}]$. Since $n^2/[n^{n+1}] \rightarrow 1$, the magnitudes compare with $1/n^2$. The signs alternate, so the left endpoint converges absolutely.
3. At $x=-1$, substitution gives $\sum 10/[n^{n+1}]$. Since $n^2/[n^{n+1}] \rightarrow 1$, the magnitudes compare with $1/n^2$. The terms are positive, so the right endpoint converges.
4. Therefore the interval is $[-5, -1]$.

4. Use $\sum_{n=1}^{\infty} \frac{4}{6^n} (x-1)^n$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-5$ and $x=7$.

Answer: Resulting series: $\sum_{n=1}^{\infty} \frac{4}{6^n} (x-1)^n$. Radius $R=6$; interval $(-5, 7)$.

1. The ratio/root calculation gives $|x-1| < 6$, so the radius is 6.
2. At $x=-5$, substitution gives $\sum 4(-1)^n/n^0$. The magnitudes are constant multiples of $1/n^0$. The signs alternate, so the left endpoint diverges because its terms do not approach 0.
3. At $x=7$, substitution gives $\sum 4/n^0$. The magnitudes are constant multiples of $1/n^0$. The terms are positive, so the right endpoint diverges because its terms do not approach 0.
4. Therefore the interval is $(-5, 7)$.

Power Series Radius and Interval of Convergence

Worked Answer Key

Find the radius first, then test the left and right endpoints separately. Recheck endpoint behavior after termwise differentiation or integration.

5. Differentiate $\sum_{n=1}^{\infty} \frac{5(7^n n^2)(x-2)^n}{7^n n^2}$ term by term to obtain $\sum_{n=1}^{\infty} 5n(x-2)^{n-1}/(7^n n^2)$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-5$ and $x=9$.

Answer: Resulting series: $\sum_{n=1}^{\infty} \frac{5n(x-2)^{n-1}}{7^n n^2}$. Radius $R=7$; interval $[-5, 9)$.

1. The ratio/root calculation gives $|x-2| < 7$, so the radius is 7.
2. At $x=-5$, substitution gives $\sum 5(-1)^{n-1}/[7^n n^2]$. The magnitudes are constant multiples of $1/n^2$. The signs alternate, so the left endpoint converges conditionally.
3. At $x=9$, substitution gives $\sum 5/[7^n n^2]$. The magnitudes are constant multiples of $1/n^2$. The terms are positive, so the right endpoint diverges as a harmonic-type series.
4. Therefore the interval is $[-5, 9)$.

6. Integrate $\sum_{n=1}^{\infty} \frac{4n!17^n(x-0)^n}{7^n n^2(n+1)}$ term by term with constant $C=0$ to obtain $\sum_{n=1}^{\infty} \frac{4(x-0)^{n+1}}{7^n n^2(n+1)}$. Find the radius and interval of convergence of the resulting series. Show separate checks at $x=-7$ and $x=7$.

Answer: Resulting series: $\sum_{n=1}^{\infty} \frac{4(x-0)^{n+1}}{7^n n^2(n+1)}$. Radius $R=7$; interval $(-7, 7)$.

1. The ratio/root calculation gives $|x-0| < 7$, so the radius is 7.
2. At $x=-7$, substitution gives $\sum \frac{4(-1)^{n+1}}{7^n n^2(n+1)}$. Since $n^2/[n^2-1(n+1)] \rightarrow 1$, the magnitudes compare with $1/n^2$. The signs alternate, so the left endpoint diverges because its terms do not approach 0.
3. At $x=7$, substitution gives $\sum \frac{4}{7^n n^2(n+1)}$. Since $n^2/[n^2-1(n+1)] \rightarrow 1$, the magnitudes compare with $1/n^2$. The terms are positive, so the right endpoint diverges because its terms do not approach 0.
4. Therefore the interval is $(-7, 7)$.