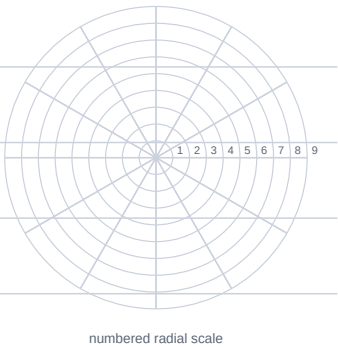


Polar Calculus

Use one complete closed-curve trace or the stated fixed spiral interval. Show $x(\theta)$, $y(\theta)$, and their derivatives for slopes; write area and arc-length integrals with correct bounds.

1. For $r = 8 \cos(3\theta)$, find or set up arc length for one petal. Use $-\pi/6 \leq \theta \leq \pi/6$ for one petal.

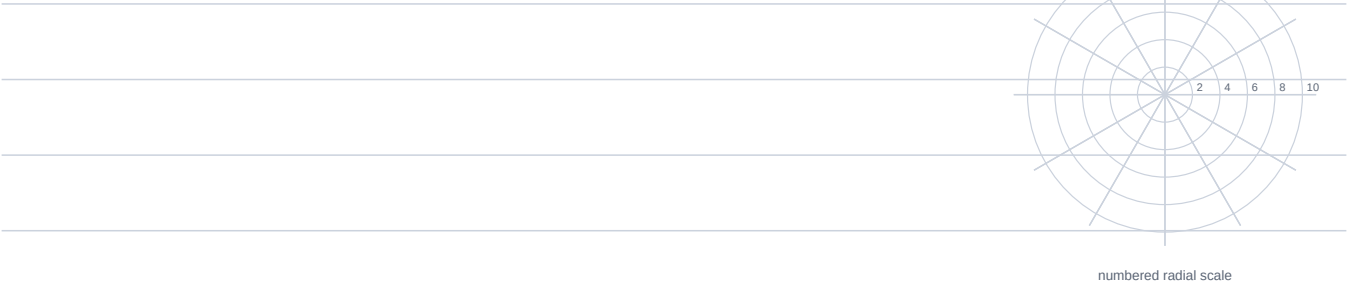


2. For $r = 9(1 + \cos(\theta))$, find dy/dx at $\theta = \pi/2$ and classify the tangent. Use exact values.

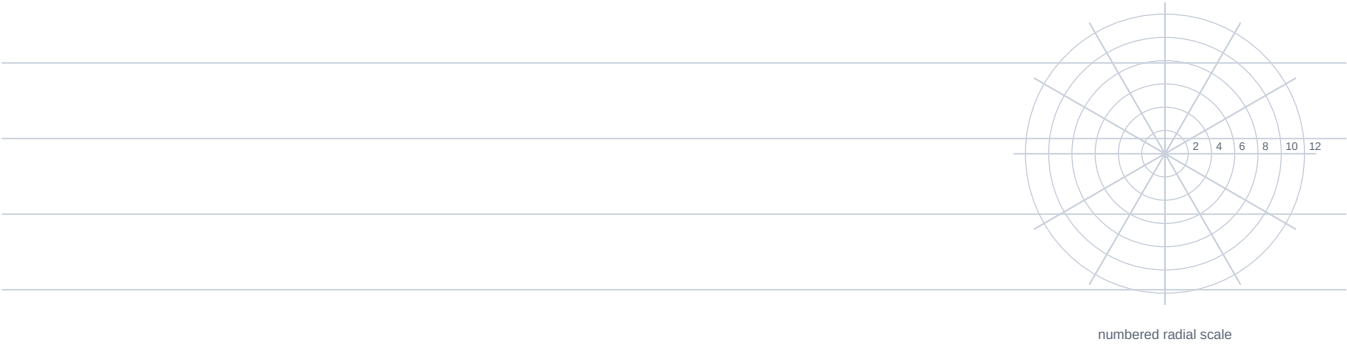
Polar Calculus

Use one complete closed-curve trace or the stated fixed spiral interval. Show $x(\theta)$, $y(\theta)$, and their derivatives for slopes; write area and arc-length integrals with correct bounds.

3. For $r = 10 \cos(\theta)$, find the area swept by one complete trace. Use $0 \leq \theta \leq \pi$.



4. For $r = 1 + 11 \cos(\theta)$, find or set up arc length over the requested nonrepeating trace. Use $0 \leq \theta \leq 2\pi$.

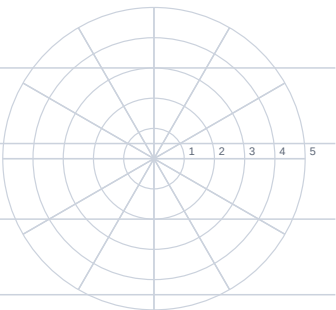


Polar Calculus

Use one complete closed-curve trace or the stated fixed spiral interval. Show $x(\theta)$, $y(\theta)$, and their derivatives for slopes; write area and arc-length integrals with correct bounds.

5. For $r = 12\theta$, find dy/dx at $\theta = \pi/2$ and classify the tangent. Use exact values.

6. For $r = 2(1 + \cos(\theta))$, find the area swept by one complete trace. Use $0 \leq \theta \leq 2\pi$.



numbered radial scale

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Polar Calculus

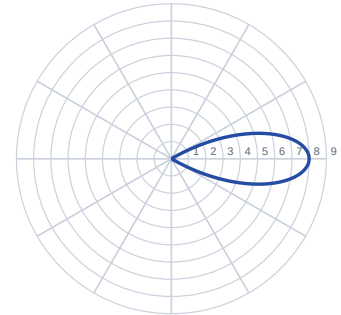
Worked Answer Key

Use one complete closed-curve trace or the stated fixed spiral interval. Show $x(\theta)$, $y(\theta)$, and their derivatives for slopes; write area and arc-length integrals with correct bounds.

1. For $r = 8 \cos(3\theta)$, find or set up arc length for one petal. Use $-\pi/6 \leq \theta \leq \pi/6$ for one petal.

Answer: integral from $-\pi/6$ to $\pi/6$ of $8\sqrt{\cos^2(3\theta) + 9\sin^2(3\theta)}$ $d\theta$.

1. Use $L = \int \sqrt{r(\theta)^2 + (dr/d\theta)^2} d\theta$ over $-\pi/6 \leq \theta \leq \pi/6$ for one petal.
2. Use bounds that do not retrace the curve.



numbered radial scale

2. For $r = 9(1 + \cos(\theta))$, find dy/dx at $\theta = \pi/2$ and classify the tangent. Use exact values.

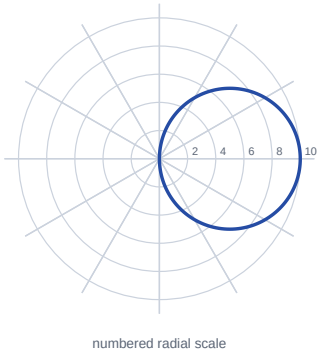
Answer: $dy/dx = 1$, so the tangent is neither horizontal nor vertical.

1. Use $dx/d\theta = r'\cos(\theta) - r\sin(\theta)$ and $dy/d\theta = r'\sin(\theta) + r\cos(\theta)$.
2. At the stated angle, $dx/d\theta = -9$ and $dy/d\theta = -9$.
3. A horizontal tangent requires $dy/d\theta = 0$ with $dx/d\theta$ nonzero; a vertical tangent requires $dx/d\theta = 0$ with $dy/d\theta$ nonzero.

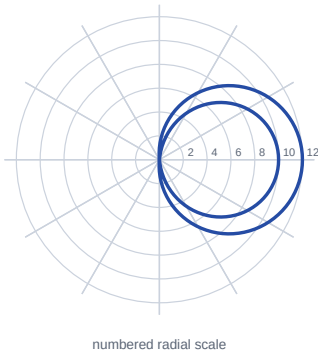
Polar Calculus
Worked Answer Key

Use one complete closed-curve trace or the stated fixed spiral interval. Show $x(\theta)$, $y(\theta)$, and their derivatives for slopes; write area and arc-length integrals with correct bounds.

3. For $r = 10 \cos(\theta)$, find the area swept by one complete trace. Use $0 \leq \theta \leq \pi$.
- Answer: $100\pi/4$.
1. Use $A = (1/2) \int r(\theta)^2 d\theta$ with bounds $0 \leq \theta \leq \pi$.
 2. The chosen bounds cover the requested region once, so no repeated-trace factor is introduced.



4. For $r = 1 + 11 \cos(\theta)$, find or set up arc length over the requested nonrepeating trace. Use $0 \leq \theta \leq 2\pi$.
- Answer: integral from 0 to 2π of $\sqrt{122 + 22\cos(\theta)}$ $d\theta$.
1. Use $L = \int \sqrt{r(\theta)^2 + (dr/d\theta)^2} d\theta$ over $0 \leq \theta \leq 2\pi$.
 2. Keep the square root nonnegative even where r is negative; arc length does not use signed radius as signed distance.



Polar Calculus

Worked Answer Key

Use one complete closed-curve trace or the stated fixed spiral interval. Show $x(\theta)$, $y(\theta)$, and their derivatives for slopes; write area and arc-length integrals with correct bounds.

5. For $r = 12\theta$, find dy/dx at $\theta = \pi/2$ and classify the tangent. Use exact values.

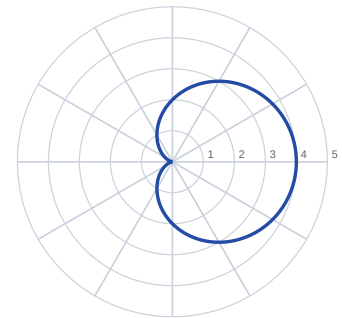
Answer: $dy/dx = -2/\pi$, so the tangent is neither horizontal nor vertical.

1. Use $dx/d\theta = r'\cos(\theta) - r\sin(\theta)$ and $dy/d\theta = r'\sin(\theta) + r\cos(\theta)$.
2. At the stated angle, $dx/d\theta = -12\pi/2$ and $dy/d\theta = 12$.
3. A horizontal tangent requires $dy/d\theta = 0$ with $dx/d\theta$ nonzero; a vertical tangent requires $dx/d\theta = 0$ with $dy/d\theta$ nonzero.

6. For $r = 2(1 + \cos(\theta))$, find the area swept by one complete trace. Use $0 \leq \theta \leq 2\pi$.

Answer: $12\pi/2$.

1. Use $A = (1/2) \int r(\theta)^2 d\theta$ with bounds $0 \leq \theta \leq 2\pi$.
2. The chosen bounds cover the requested region once, so no repeated-trace factor is introduced.



numbered radial scale