

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Partial Fraction Integration

Decompose each proper rational function, integrate each term, and preserve absolute values on linear logarithms. Definite intervals never cross a pole.

1. Evaluate the indefinite integral of $(1x+0)/(x-2)^2 dx$. A valid decomposition is $1/(x-2) + 2/(x-2)^2$.

2. Evaluate the definite integral from 0 to 1 of $((6)x-39)/((x-4)(x-7)) dx$.

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Decompose each proper rational function, integrate each term, and preserve absolute values on linear logarithms. Definite intervals never cross a pole.

3. Evaluate the indefinite integral of $(7x^2-4x+40)/((x-2)(x^2+16))$ dx. A valid decomposition is $3/(x-2) + (4x+4)/(x^2+16)$.

4. Evaluate the definite integral from 0 to 1 of $(4x^2-8x+9)/((x-4)(x^2+25))$ dx.

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Partial Fraction Integration

Decompose each proper rational function, integrate each term, and preserve absolute values on linear logarithms. Definite intervals never cross a pole.

5. Evaluate the indefinite integral of $(5x-16)/(x-4)^2 dx$. A valid decomposition is $5/(x-4) + 4/(x-4)^2$.

6. Evaluate the definite integral from 0 to 1 of $((6)x-23)/((x-3)(x-8)) dx$.

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Partial Fraction Integration

Worked Answer Key

Decompose each proper rational function, integrate each term, and preserve absolute values on linear logarithms. Definite intervals never cross a pole.

1. Evaluate the indefinite integral of $(1x+0)/(x-2)^2$ dx. A valid decomposition is $1/(x-2) + 2/(x-2)^2$.

Answer: $1 \ln|x-2| - 2/(x-2) + C$

1. Use $1/(x-2) + 2/(x-2)^2$; recombination verifies the numerator.
2. Integrate termwise: $1 \ln|x-2| - 2/(x-2)$.
3. Keep absolute values on linear logarithms and include the integration constant.

2. Evaluate the definite integral from 0 to 1 of $((6)x-39)/((x-4)(x-7))$ dx.

Answer: $5 \ln(3/4) + 1 \ln(6/7)$

1. Set the decomposition to $5/(x-4) + 1/(x-7)$. Equating coefficients gives $A = 5$, $B = 1$.
2. Integrate termwise: $5 \ln|x-4| + 1 \ln|x-7|$.
3. Neither denominator factor vanishes on $[0,1]$. Subtract $F(0)$ from $F(1)$: $5 \ln(3/4) + 1 \ln(6/7)$.

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Partial Fraction Integration

Worked Answer Key

Decompose each proper rational function, integrate each term, and preserve absolute values on linear logarithms. Definite intervals never cross a pole.

3. Evaluate the indefinite integral of $(7x^2-4x+40)/((x-2)(x^2+16))$ dx. A valid decomposition is $3/(x-2) + (4x+4)/(x^2+16)$.

Answer: $3 \ln|x-2| + 2 \ln(x^2+16) + 1 \operatorname{atan}(x/4) + C$

1. Use $3/(x-2) + (4x+4)/(x^2+16)$; recombination verifies the numerator.
2. Integrate termwise: $3 \ln|x-2| + 2 \ln(x^2+16) + 1 \operatorname{atan}(x/4)$.
3. Keep absolute values on linear logarithms and include the integration constant.

4. Evaluate the definite integral from 0 to 1 of $(4x^2-8x+9)/((x-4)(x^2+25))$ dx.

Answer: $1 \ln(3/4) + 3/2 \ln(26/25) + 4/5 \operatorname{atan}(1/5)$

1. Set the decomposition to $1/(x-4) + (3x+4)/(x^2+25)$. $A(x^2+25)+(3x+4)(x-4)$ reproduces the numerator, so $A = 1$, $B = 3$, $C = 4$.
2. Integrate termwise: $1 \ln|x-4| + 3/2 \ln(x^2+25) + 4/5 \operatorname{atan}(x/5)$.
3. Neither denominator factor vanishes on $[0,1]$. Subtract $F(0)$ from $F(1)$: $1 \ln(3/4) + 3/2 \ln(26/25) + 4/5 \operatorname{atan}(1/5)$.

Partial Fraction Integration

Worked Answer Key

Decompose each proper rational function, integrate each term, and preserve absolute values on linear logarithms. Definite intervals never cross a pole.

5. Evaluate the indefinite integral of $(5x-16)/(x-4)^2 dx$. A valid decomposition is $5/(x-4) + 4/(x-4)^2$.

Answer: $5 \ln|x-4| - 4/(x-4) + C$

1. Use $5/(x-4) + 4/(x-4)^2$; recombination verifies the numerator.
2. Integrate termwise: $5 \ln|x-4| - 4/(x-4)$.
3. Keep absolute values on linear logarithms and include the integration constant.

6. Evaluate the definite integral from 0 to 1 of $((6)x-23)/((x-3)(x-8)) dx$.

Answer: $1 \ln(2/3) + 5 \ln(7/8)$

1. Set the decomposition to $1/(x-3) + 5/(x-8)$. Equating coefficients gives $A = 1$, $B = 5$.
2. Integrate termwise: $1 \ln|x-3| + 5 \ln|x-8|$.
3. Neither denominator factor vanishes on $[0,1]$. Subtract $F(0)$ from $F(1)$: $1 \ln(2/3) + 5 \ln(7/8)$.