

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Parametric Calculus

Show derivatives with respect to t before forming dy/dx . Check dx/dt and dy/dt when classifying tangents, and preserve every stated traversal interval.

1. For $x = 4 \cos(t)$, $y = 4 \sin(t)$, find d^2y/dx^2 where $\sin(t) \neq 0$.

2. For $x = 3t + 2$, $y = t^2 + 3t$, find dy/dx and the tangent line at $t = 3$.

Parametric Calculus

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3. For the parametric curve shown by $x = 6 \cos(t)$, $y = 6 \sin(t)$, $0 \leq t \leq 2\pi$, make a value table and sketch the traced curve with arrows for increasing t.

t	0	π	2π
(x,y)			



4. Find horizontal and vertical tangents for $x = t^2 + 0$, $y = t^3 - 108t$. Verify the companion derivative is nonzero.

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Parametric Calculus

Show derivatives with respect to t before forming dy/dx . Check dx/dt and dy/dt when classifying tangents, and preserve every stated traversal interval.

5. The arc $x = t$, $y = 21(1 - t^2)$, $-1 \leq t \leq 1$, is traversed left to right and closed by the x -axis. Find the enclosed area with correct bounds and orientation.

6. For $x = 9 \cos(t)$, $y = -9 \sin(t)$, find speed and arc length on $0 \leq t \leq 4\pi/2$.

Parametric Calculus

Worked Answer Key

Show derivatives with respect to t before forming dy/dx . Check dx/dt and dy/dt when classifying tangents, and preserve every stated traversal interval.

- 1.** For $x = 4 \cos(t)$, $y = 4 \sin(t)$, find d^2y/dx^2 where $\sin(t) \neq 0$.

Answer: $d^2y/dx^2 = -1/(4 \sin^3(t))$.

1. First, $dy/dx = -\cot(t)$. Differentiate this slope with respect to t .
2. Divide $d(dy/dx)/dt$ by $dx/dt = -4 \sin(t)$; the condition $\sin(t) \neq 0$ keeps dx/dt nonzero.

- 2.** For $x = 3t + 2$, $y = t^2 + 3t$, find dy/dx and the tangent line at $t = 3$.

Answer: $dy/dx = (2t + 3)/3$; at $t = 3$, $m = 9/3$, so $y - 18 = (9/3)(x - 11)$.

1. $dx/dt = 3$; $dy/dt = 2t + 3$.
2. Because dx/dt is nonzero, divide and use the parametric point at t_0 in point-slope form.

Parametric Calculus
Worked Answer Key

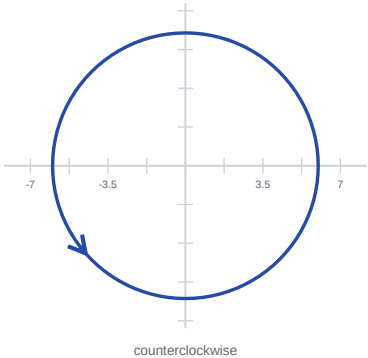
Show derivatives with respect to t before forming dy/dx. Check dx/dt and dy/dt when classifying tangents, and preserve every stated traversal interval.

3. For the parametric curve shown by $x = 6 \cos(t)$, $y = 6 \sin(t)$, $0 \leq t \leq 2\pi$, make a value table and sketch the traced curve with arrows for increasing t.

t	0	π	2π
(x,y)	(6, 0)	(-6, 0)	(6, 0)

Answer: A circle is traced once; orientation is counterclockwise.

- 1. Evaluate x(t) and y(t) at the interval endpoints and central values.
- 2. Connect points in increasing-t order; the parameter interval determines the retained curve.



4. Find horizontal and vertical tangents for $x = t^2 + 0$, $y = t^3 - 108t$. Verify the companion derivative is nonzero.

Answer: Horizontal at $t = -6$ and 6 ; vertical at $t = 0$.

- 1. $dy/dt = 3t^2 - 108 = 0$ at $t = \pm 6$, and $dx/dt = 2t$ is nonzero there.
- 2. $dx/dt = 2t = 0$ at $t = 0$, and $dy/dt = -108$ is nonzero there.

Parametric Calculus

Worked Answer Key

Show derivatives with respect to t before forming dy/dx . Check dx/dt and dy/dt when classifying tangents, and preserve every stated traversal interval.

5. The arc $x = t$, $y = 21(1 - t^2)$, $-1 \leq t \leq 1$, is traversed left to right and closed by the x -axis. Find the enclosed area with correct bounds and orientation.

Answer: Area = 28.

1. Because $dx/dt = 1$ and $y \geq 0$, $A = \text{integral from } -1 \text{ to } 1 \text{ of } 21(1 - t^2) dt$.
2. The left-to-right orientation makes integral $y dx$ positive; reversing the traversal would change the signed integral, not the geometric area.

6. For $x = 9 \cos(t)$, $y = -9 \sin(t)$, find speed and arc length on $0 \leq t \leq 4\pi/2$.

Answer: Speed = 9; length = $36\pi/2$.

1. Speed = $\sqrt{(dx/dt)^2 + (dy/dt)^2} = \sqrt{81(\sin^2(t) + \cos^2(t))} = 9$.
2. Integrate the nonnegative speed from 0 to $4\pi/2$; this interval does not repeat the circle.