

Optimization

Define the feasible domain, maximize the objective, and report dimensions with units.

1. rectangle

A rectangle has perimeter 32 ft. Find the dimensions that maximize its area.

Set-up and work:



x
16 - x

2. rectangle

A rectangle has perimeter 24 cm. Find the dimensions that maximize its area.

Set-up and work:



x
12 - x

Optimization

Define the feasible domain, maximize the objective, and report dimensions with units.

3. three side fence

A rectangular enclosure borders a river, so only three sides need 40 ft of fencing. Find the dimensions that maximize area.

Set-up and work:

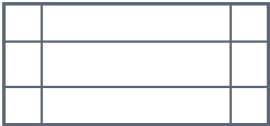


40 - 2x
x
river: no fence

4. open top box

Squares of side x are cut from each corner of a 24 cm square sheet and the sides are folded up. Find the cut that maximizes box volume.

Set-up and work:



x
24 - 2x

Optimization

Worked Answer Key

Define the feasible domain, maximize the objective, and report dimensions with units.

1. rectangle

A rectangle has perimeter 32 ft. Find the dimensions that maximize its area.

Constraint:

$$2 \cdot x + 2 \cdot y = 32$$

Objective:

$$A = 16 \cdot x - x^2$$

Derivative:

$$A' = 16 - 2 \cdot x$$

Feasible domain: $0 \leq x \leq 16$.

The endpoints give area 0, while the only interior critical point gives the positive area shown; therefore it is the global maximum on the closed feasible interval.

Answer: 8 by 8 ft; maximum area 64 ft².

Optimization

Worked Answer Key

Define the feasible domain, maximize the objective, and report dimensions with units.

2. rectangle

A rectangle has perimeter 24 cm. Find the dimensions that maximize its area.

Constraint:

$$2 \cdot x + 2 \cdot y = 24$$

Objective:

$$A = 12 \cdot x - x^2$$

Derivative:

$$A' = 12 - 2 \cdot x$$

Feasible domain: $0 \leq x \leq 12$.

The endpoints give area 0, while the only interior critical point gives the positive area shown; therefore it is the global maximum on the closed feasible interval.

Answer: 6 by 6 cm; maximum area 36 cm².

Optimization

Worked Answer Key

Define the feasible domain, maximize the objective, and report dimensions with units.

3. three side fence

A rectangular enclosure borders a river, so only three sides need 40 ft of fencing. Find the dimensions that maximize area.

Constraint:

$$y + 2 \cdot x = 40$$

Objective:

$$A = 40 \cdot x - 2 \cdot x^2$$

Derivative:

$$A' = 40 - 4 \cdot x$$

Feasible domain: $0 \leq x \leq 20$.

The endpoint areas are 0 and the concave-down objective has its only interior critical point here, so this is the feasible global maximum.

Answer: 20 by 10 ft; maximum area 200 ft².

Optimization

Worked Answer Key

Define the feasible domain, maximize the objective, and report dimensions with units.

4. open top box

Squares of side x are cut from each corner of a 24 cm square sheet and the sides are folded up. Find the cut that maximizes box volume.

Objective:

$$V = x \cdot (24 - 2 \cdot x)^2$$

Derivative:

$$V' = 576 - 192 \cdot x + 12 \cdot x^2$$

Critical cut:

$$x = 4$$

Feasible domain: $0 \leq x \leq 12$.

The derivative factors as $(L - 2x)(L - 6x)$. On $0 \leq x \leq L/2$, $x = L/6$ is the only interior maximum candidate; both endpoints give volume 0, proving the global maximum.

Answer: 16 by 16 by 4 cm; maximum volume 1024 cm^3 .