

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

1. index 6

Approximate $\sqrt[6]{93}$; $f(x) = x^6 - 93$

Start $x_0 = 2.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

2. index 3

Approximate $\sqrt[3]{498}$; $f(x) = x^3 - 498$

Start $x_0 = 8.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

3. index 5

Approximate $\sqrt[5]{303}$; $f(x) = x^5 - 303$

Start $x_0 = 3.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

4. index 4

Approximate $\sqrt[4]{465}$; $f(x) = x^4 - 465$

Start $x_0 = 5.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

5. index 2

Approximate $\sqrt{381}$; $f(x) = x^2 - 381$

Start $x_0 = 20.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

6. index 6

Approximate $\sqrt[6]{285}$; $f(x) = x^6 - 285$

Start $x_0 = 3.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

7. index 5

Approximate $\sqrt[5]{435}$; $f(x) = x^5 - 435$

Start $x_0 = 3.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

8. index 3

Approximate $\sqrt[3]{60}$; $f(x) = x^3 - 60$

Start $x_0 = 4.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

9. index 4

Approximate $\sqrt[4]{431}$; $f(x) = x^4 - 431$

Start $x_0 = 5.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

10. index 2

Approximate $\sqrt{185}$; $f(x) = x^2 - 185$

Start $x_0 = 14.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

11. index 5

Approximate $\sqrt[5]{340}$; $f(x) = x^5 - 340$

Start $x_0 = 3.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

12. index 6

Approximate $\sqrt[6]{71}$; $f(x) = x^6 - 71$

Start $x_0 = 2.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

13. index 2

Approximate $\sqrt{412}$; $f(x) = x^2 - 412$

Start $x_0 = 20.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

14. index 3

Approximate $\sqrt[3]{22}$; $f(x) = x^3 - 22$

Start $x_0 = 3.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

15. index 4

Approximate $\sqrt[4]{420}$; $f(x) = x^4 - 420$

Start $x_0 = 5.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

16. index 3

Approximate $\sqrt[3]{455}$; $f(x) = x^3 - 455$

Start $x_0 = 8.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

17. index 5

Approximate $\sqrt[5]{308}$; $f(x) = x^5 - 308$

Start $x_0 = 3.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

18. index 6

Approximate $\sqrt[6]{205}$; $f(x) = x^6 - 205$

Start $x_0 = 2.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

19. index 2

Approximate $\sqrt{438}$; $f(x) = x^2 - 438$

Start $x_0 = 21.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

20. index 4

Approximate $\sqrt[4]{136}$; $f(x) = x^4 - 136$

Start $x_0 = 3.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

21. index 5

Approximate $\sqrt[5]{269}$; $f(x) = x^5 - 269$

Start $x_0 = 3.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Newton's Method for Roots

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

22. index 4

Approximate $\sqrt[4]{382}$; $f(x) = x^4 - 382$

Start $x_0 = 4.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

23. index 2

Approximate $\sqrt{301}$; $f(x) = x^2 - 301$

Start $x_0 = 17.500000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

24. index 3

Approximate $\sqrt[3]{25}$; $f(x) = x^3 - 25$

Start $x_0 = 3.000000$. Complete 2 Newton iterations using full precision.

Derivative $f'(x)$: _____

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{(i+1)}$
0	_____	_____	_____	_____
1	_____	_____	_____	_____

Nth iterate: _____

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

1. index 6

$f(x) = x^6 - 93$

$f'(x) = 6 \cdot x^5$

$x_{next} = x - \frac{x^6 - 93}{6 \cdot x^5}$

root update $= \frac{5 \cdot x + \frac{93}{x^5}}{6}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	2.500000	151.140625	585.937500	0.257947	2.242053
1	2.242053	34.021032	339.923310	0.100084	2.141969

After 2 iterations, x_2 approximately equals 2.141969. The residual |f(x_2)| is approximately 3.578169; this reports the computed approximation, not an exact zero.

2. index 3

$f(x) = x^3 - 498$

$f'(x) = 3 \cdot x^2$

$x_{next} = x - \frac{x^3 - 498}{3 \cdot x^2}$

root update $= \frac{2 \cdot x + \frac{498}{x^2}}{3}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	8.000000	14.000000	192.000000	0.072917	7.927083
1	7.927083	0.127216	188.515951	0.000675	7.926409

After 2 iterations, x_2 approximately equals 7.926409. The residual |f(x_2)| is approximately 1.082961e-5; this reports the computed approximation, not an exact zero.

Newton's Method for Roots

Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

3. index 5

$$f(x) = x^5 - 303$$

$$f'(x) = 5 \cdot x^4$$

$$x_{\text{next}} = x - \frac{x^5 - 303}{5 \cdot x^4}$$

$$\text{root update} = \frac{4 \cdot x + \frac{303}{x^4}}{5}$$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.500000	222.218750	750.312500	0.296168	3.203832
1	3.203832	34.558070	526.803681	0.065600	3.138232

After 2 iterations, x_2 approximately equals 3.138232. The residual $|f(x_2)|$ is approximately 1.386495; this reports the computed approximation, not an exact zero.

4. index 4

$$f(x) = x^4 - 465$$

$$f'(x) = 4 \cdot x^3$$

$$x_{\text{next}} = x - \frac{x^4 - 465}{4 \cdot x^3}$$

$$\text{root update} = \frac{3 \cdot x + \frac{465}{x^3}}{4}$$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	5.000000	160.000000	500.000000	0.320000	4.680000
1	4.680000	14.715126	410.012928	0.035889	4.644111

After 2 iterations, x_2 approximately equals 4.644111. The residual $|f(x_2)|$ is approximately 0.168405; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

5. index 2

$f(x) = x^2 - 381$
 $f'(x) = 2 \cdot x^1$
 $x_{next} = x - \frac{x^2 - 381}{2 \cdot x^1}$
 $root\ update = \frac{x + \frac{381}{x^1}}{2}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	20.000000	19.000000	40.000000	0.475000	19.525000
1	19.525000	0.225625	39.050000	0.005778	19.519222

After 2 iterations, x_2 approximately equals 19.519222. The residual |f(x_2)| is approximately 3.338354e-5; this reports the computed approximation, not an exact zero.

6. index 6

$f(x) = x^6 - 285$
 $f'(x) = 6 \cdot x^5$
 $x_{next} = x - \frac{x^6 - 285}{6 \cdot x^5}$
 $root\ update = \frac{5 \cdot x + \frac{285}{x^5}}{6}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.000000	444.000000	1458.000000	0.304527	2.695473
1	2.695473	98.539554	853.741479	0.115421	2.580052

After 2 iterations, x_2 approximately equals 2.580052. The residual |f(x_2)| is approximately 9.965468; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

7. index 5

$f(x) = x^5 - 435$

$f'(x) = 5 \cdot x^4$

$x_{next} = x - \frac{x^5 - 435}{5 \cdot x^4}$

root update $= \frac{4 \cdot x + \frac{435}{x^4}}{5}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.500000	90.218750	750.312500	0.120242	3.379758
1	3.379758	5.989554	652.398038	0.009181	3.370578

After 2 iterations, x_2 approximately equals 3.370578. The residual |f(x_2)| is approximately 0.032452; this reports the computed approximation, not an exact zero.

8. index 3

$f(x) = x^3 - 60$

$f'(x) = 3 \cdot x^2$

$x_{next} = x - \frac{x^3 - 60}{3 \cdot x^2}$

root update $= \frac{2 \cdot x + \frac{60}{x^2}}{3}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	4.000000	4.000000	48.000000	0.083333	3.916667
1	3.916667	0.082755	46.020833	0.001798	3.914868

After 2 iterations, x_2 approximately equals 3.914868. The residual |f(x_2)| is approximately 3.798805e-5; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

9. index 4

$f(x) = x^4 - 431$

$f'(x) = 4 \cdot x^3$

$x_{next} = x - \frac{x^4 - 431}{4 \cdot x^3}$

$root\ update = \frac{3 \cdot x + \frac{431}{x^3}}{4}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	5.000000	194.000000	500.000000	0.388000	4.612000
1	4.612000	21.436042	392.398996	0.054628	4.557372

After 2 iterations, x_2 approximately equals 4.557372. The residual |f(x_2)| is approximately 0.377860; this reports the computed approximation, not an exact zero.

10. index 2

$f(x) = x^2 - 185$

$f'(x) = 2 \cdot x^1$

$x_{next} = x - \frac{x^2 - 185}{2 \cdot x^1}$

$root\ update = \frac{x + \frac{185}{x^1}}{2}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	14.000000	11.000000	28.000000	0.392857	13.607143
1	13.607143	0.154337	27.214286	0.005671	13.601472

After 2 iterations, x_2 approximately equals 13.601472. The residual |f(x_2)| is approximately 3.216212e-5; this reports the computed approximation, not an exact zero.

Newton's Method for Roots

Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

11. index 5

$$f(x) = x^5 - 340$$

$$f'(x) = 5 \cdot x^4$$

$$x_{\text{next}} = x - \frac{x^5 - 340}{5 \cdot x^4}$$

$$\text{root update} = \frac{4 \cdot x + \frac{340}{x^4}}{5}$$

i	x _i	f(x _i)	f'(x _i)	correction	x _(i+1)
0	3.500000	185.218750	750.312500	0.246855	3.253145
1	3.253145	24.348334	559.994078	0.043480	3.209665

After 2 iterations, x₂ approximately equals 3.209665. The residual |f(x₂)| is approximately 0.642210; this reports the computed approximation, not an exact zero.

12. index 6

$$f(x) = x^6 - 71$$

$$f'(x) = 6 \cdot x^5$$

$$x_{\text{next}} = x - \frac{x^6 - 71}{6 \cdot x^5}$$

$$\text{root update} = \frac{5 \cdot x + \frac{71}{x^5}}{6}$$

i	x _i	f(x _i)	f'(x _i)	correction	x _(i+1)
0	2.500000	173.140625	585.937500	0.295493	2.204507
1	2.204507	43.780602	312.398063	0.140144	2.064363

After 2 iterations, x₂ approximately equals 2.064363. The residual |f(x₂)| is approximately 6.395628; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

13. index 2

$f(x) = x^2 - 412$

$f'(x) = 2 \cdot x^1$

$x_{next} = x - \frac{x^2 - 412}{2 \cdot x^1}$

$root\ update = \frac{x + \frac{412}{x^1}}{2}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	20.500000	8.250000	41.000000	0.201220	20.298780
1	20.298780	0.040489	40.597561	0.000997	20.297783

After 2 iterations, x_2 approximately equals 20.297783. The residual |f(x_2)| is approximately 9.946734e-7; this reports the computed approximation, not an exact zero.

14. index 3

$f(x) = x^3 - 22$

$f'(x) = 3 \cdot x^2$

$x_{next} = x - \frac{x^3 - 22}{3 \cdot x^2}$

$root\ update = \frac{2 \cdot x + \frac{22}{x^2}}{3}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.000000	5.000000	27.000000	0.185185	2.814815
1	2.814815	0.302291	23.769547	0.012718	2.802097

After 2 iterations, x_2 approximately equals 2.802097. The residual |f(x_2)| is approximately 0.001364; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

15. index 4

$f(x) = x^4 - 420$

$f'(x) = 4 \cdot x^3$

$x_{next} = x - \frac{x^4 - 420}{4 \cdot x^3}$

root update $= \frac{3 \cdot x + \frac{420}{x^3}}{4}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	5.000000	205.000000	500.000000	0.410000	4.590000
1	4.590000	23.864838	386.810316	0.061696	4.528304

After 2 iterations, x_2 approximately equals 4.528304. The residual |f(x_2)| is approximately 0.476872; this reports the computed approximation, not an exact zero.

16. index 3

$f(x) = x^3 - 455$

$f'(x) = 3 \cdot x^2$

$x_{next} = x - \frac{x^3 - 455}{3 \cdot x^2}$

root update $= \frac{2 \cdot x + \frac{455}{x^2}}{3}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	8.000000	57.000000	192.000000	0.296875	7.703125
1	7.703125	2.089069	178.014404	0.011735	7.691390

After 2 iterations, x_2 approximately equals 7.691390. The residual |f(x_2)| is approximately 0.003181; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

17. index 5

$f(x) = x^5 - 308$

$f'(x) = 5 \cdot x^4$

$x_{next} = x - \frac{x^5 - 308}{5 \cdot x^4}$

root update $= \frac{4 \cdot x + \frac{308}{x^4}}{5}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.500000	217.218750	750.312500	0.289504	3.210496
1	3.210496	33.083266	531.200329	0.062280	3.148215

After 2 iterations, x_2 approximately equals 3.148215. The residual |f(x_2)| is approximately 1.258901; this reports the computed approximation, not an exact zero.

18. index 6

$f(x) = x^6 - 205$

$f'(x) = 6 \cdot x^5$

$x_{next} = x - \frac{x^6 - 205}{6 \cdot x^5}$

root update $= \frac{5 \cdot x + \frac{205}{x^5}}{6}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	2.500000	39.140625	585.937500	0.066800	2.433200
1	2.433200	2.523291	511.729306	0.004931	2.428269

After 2 iterations, x_2 approximately equals 2.428269. The residual |f(x_2)| is approximately 0.012749; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

19. index 2

$f(x) = x^2 - 438$

$f'(x) = 2 \cdot x^1$

$x_{next} = x - \frac{x^2 - 438}{2 \cdot x^1}$

$root\ update = \frac{x + \frac{438}{x^1}}{2}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	21.000000	3.000000	42.000000	0.071429	20.928571
1	20.928571	0.005102	41.857143	0.000122	20.928450

After 2 iterations, x_2 approximately equals 20.928450. The residual |f(x_2)| is approximately 1.485756e-8; this reports the computed approximation, not an exact zero.

20. index 4

$f(x) = x^4 - 136$

$f'(x) = 4 \cdot x^3$

$x_{next} = x - \frac{x^4 - 136}{4 \cdot x^3}$

$root\ update = \frac{3 \cdot x + \frac{136}{x^3}}{4}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.500000	14.062500	171.500000	0.081997	3.418003
1	3.418003	0.486506	159.726611	0.003046	3.414957

After 2 iterations, x_2 approximately equals 3.414957. The residual |f(x_2)| is approximately 0.000650; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

21. index 5

$f(x) = x^5 - 269$

$f'(x) = 5 \cdot x^4$

$x_{next} = x - \frac{x^5 - 269}{5 \cdot x^4}$

root update $= \frac{4 \cdot x + \frac{269}{x^4}}{5}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.500000	256.218750	750.312500	0.341483	3.158517
1	3.158517	45.352044	497.625968	0.091137	3.067380

After 2 iterations, x_2 approximately equals 3.067380. The residual |f(x_2)| is approximately 2.542769; this reports the computed approximation, not an exact zero.

22. index 4

$f(x) = x^4 - 382$

$f'(x) = 4 \cdot x^3$

$x_{next} = x - \frac{x^4 - 382}{4 \cdot x^3}$

root update $= \frac{3 \cdot x + \frac{382}{x^3}}{4}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	4.500000	28.062500	364.500000	0.076989	4.423011
1	4.423011	0.711989	346.109916	0.002057	4.420954

After 2 iterations, x_2 approximately equals 4.420954. The residual |f(x_2)| is approximately 0.000497; this reports the computed approximation, not an exact zero.

Newton's Method for Roots
Worked Answer Key

Use the stated positive starting value and Newton update to approximate each indexed radical. Carry full precision between iterations.

23. index 2

$f(x) = x^2 - 301$

$f'(x) = 2 \cdot x^1$

$x_{next} = x - \frac{x^2 - 301}{2 \cdot x^1}$

$root\ update = \frac{x + \frac{301}{x^1}}{2}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	17.500000	5.250000	35.000000	0.150000	17.350000
1	17.350000	0.022500	34.700000	0.000648	17.349352

After 2 iterations, x_2 approximately equals 17.349352. The residual |f(x_2)| is approximately 4.204421e-7; this reports the computed approximation, not an exact zero.

24. index 3

$f(x) = x^3 - 25$

$f'(x) = 3 \cdot x^2$

$x_{next} = x - \frac{x^3 - 25}{3 \cdot x^2}$

$root\ update = \frac{2 \cdot x + \frac{25}{x^2}}{3}$

i	x_i	f(x_i)	f'(x_i)	correction	x_(i+1)
0	3.000000	2.000000	27.000000	0.074074	2.925926
1	2.925926	0.048976	25.683128	0.001907	2.924019

After 2 iterations, x_2 approximately equals 2.924019. The residual |f(x_2)| is approximately 3.191287e-5; this reports the computed approximation, not an exact zero.