

Logarithmic Differentiation

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

1. Differentiate using logarithmic differentiation.

$$y = 2 \cdot (2 \cdot x^2 + 3)^{2 \cdot x}$$

Required base domain: $2 \cdot (x)^2 + 3 > 0$.

2. Differentiate using logarithmic differentiation.

$$y = -2 \cdot (3 \cdot x^2 + 1)^{\frac{4}{3}} \cdot (3 \cdot x - 1)^{\frac{3}{2}} \cdot (-1 \cdot x + 5)^{\frac{3}{2}}$$

Show the logarithm step and retain every original denominator exclusion.

Logarithmic Differentiation

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

3. Differentiate using logarithmic differentiation.

$$y = \frac{(2 \cdot x)^{\frac{2}{3}}}{(-1 \cdot x + 4)^2}$$

Show the logarithm step and retain every original denominator exclusion.

4. Differentiate using logarithmic differentiation.

$$y = -3 \cdot (2 \cdot x + 5)^{-1} \cdot x + 2$$

Required base domain: $2 \cdot x + 5 > 0$.

Logarithmic Differentiation

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

5. Differentiate using logarithmic differentiation.

$$y = \left(2 \cdot x^2 + 1\right)^{\frac{3}{2}} \cdot (-3 \cdot x + 4)^3$$

Show the logarithm step and retain every original denominator exclusion.

6. Differentiate using logarithmic differentiation.

$$y = \frac{-2 \cdot (-2 \cdot x + 4)^3 \cdot (2 \cdot x + 4)^3}{\left(3 \cdot x^2 + 1\right)^{\frac{1}{2}}}$$

Show the logarithm step and retain every original denominator exclusion.

Logarithmic Differentiation

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

7. Differentiate using logarithmic differentiation.

$$y = 2 \cdot (x^2 + 3)^{x+1}$$

Required base domain: $(x)^{(2)} + 3 > 0$.

8. Differentiate using logarithmic differentiation.

$$y = -2 \cdot (2 \cdot x^2 + 2)^{\frac{3}{2}} \cdot (3 \cdot x + 2)^{\frac{1}{2}} \cdot (-2 \cdot x + 5)^3$$

Show the logarithm step and retain every original denominator exclusion.

Logarithmic Differentiation

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

9. Differentiate using logarithmic differentiation.

$$y = \frac{(2 \cdot x + 3)^{\frac{2}{3}}}{(-2 \cdot x + 5)^{\frac{1}{3}}}$$

Show the logarithm step and retain every original denominator exclusion.

10. Differentiate using logarithmic differentiation.

$$y = -3 \cdot (x + 5)^{-3 \cdot x + 1}$$

Required base domain: $x + 5 > 0$.

Logarithmic Differentiation

Worked Answer Key

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

1. $y =$

$$2 \cdot (2 \cdot x^2 + 3)^{2 \cdot x}$$

$$\ln|y| = \ln(2) + 2 \cdot x \cdot \ln(2 \cdot x^2 + 3)$$

$$(dy/dx)/y = 2 \cdot \ln(2 \cdot x^2 + 3) + 2 \cdot x \cdot \frac{4 \cdot x}{2 \cdot x^2 + 3}$$

$$\text{Answer: } dy/dx = 2 \cdot (2 \cdot x^2 + 3)^{2 \cdot x} \cdot \left(2 \cdot \ln(2 \cdot x^2 + 3) + 2 \cdot x \cdot \frac{4 \cdot x}{2 \cdot x^2 + 3} \right)$$

Original domain: $2 \cdot (x)^2 + 3 > 0$. Logarithmic formula: $2 \cdot (x)^2 + 3 > 0$.

2. $y =$

$$-2 \cdot (3 \cdot x^2 + 1)^{\frac{4}{3}} \cdot (3 \cdot x - 1)^{\frac{3}{2}} \cdot (-1 \cdot x + 5)^{\frac{3}{2}}$$

$$\ln|y| = \ln(-2) + \frac{4}{3} \cdot \ln(3 \cdot x^2 + 1) + \frac{3}{2} \cdot \ln(3 \cdot x - 1) + \frac{3}{2} \cdot \ln(-1 \cdot x + 5)$$

$$(dy/dx)/y = \frac{4}{3} \cdot \frac{6 \cdot x}{3 \cdot x^2 + 1} + \frac{3}{2} \cdot \frac{3}{3 \cdot x - 1} + \frac{3}{2} \cdot \frac{-1}{-1 \cdot x + 5}$$

$$\text{Answer: } dy/dx = -2 \cdot (3 \cdot x^2 + 1)^{\frac{4}{3}} \cdot (3 \cdot x - 1)^{\frac{3}{2}} \cdot (-1 \cdot x + 5)^{\frac{3}{2}} \cdot \left(\frac{4}{3} \cdot \frac{6 \cdot x}{3 \cdot x^2 + 1} + \frac{3}{2} \cdot \frac{3}{3 \cdot x - 1} + \frac{3}{2} \cdot \frac{-1}{-1 \cdot x + 5} \right)$$

Original domain: $3 \cdot x - 1 \geq 0$; $-1 \cdot x + 5 \geq 0$. Logarithmic formula: $3 \cdot (x)^2 + 1 \neq 0$; $3 \cdot x - 1 > 0$; $-1 \cdot x + 5 > 0$.

At $x = 1/3$, the point is a one-sided domain endpoint. At $x = 5$, the point is a one-sided domain endpoint.

3. $y =$

$$\frac{(2 \cdot x)^{\frac{2}{3}}}{(-1 \cdot x + 4)^2}$$

$$\ln|y| = \ln(2) + \frac{2}{3} \cdot \ln(2 \cdot x) - 2 \cdot \ln(-1 \cdot x + 4)$$

$$(dy/dx)/y = \frac{2}{3} \cdot \frac{2}{2 \cdot x} - 2 \cdot \frac{-1}{-1 \cdot x + 4}$$

$$\text{Answer: } dy/dx = \frac{(2 \cdot x)^{\frac{2}{3}}}{(-1 \cdot x + 4)^2} \cdot \left(\frac{2}{3} \cdot \frac{2}{2 \cdot x} - 2 \cdot \frac{-1}{-1 \cdot x + 4} \right)$$

Original domain: $-1 \cdot x + 4 \neq 0$. Logarithmic formula: $2 \cdot x \neq 0$; $-1 \cdot x + 4 \neq 0$.

At $x = 0$, no finite two-sided derivative exists.

4. $y =$

$$-3 \cdot (2 \cdot x + 5)^{-1 \cdot x + 2}$$

$$\ln|y| = \ln(-3) + (-1 \cdot x + 2) \cdot \ln(2 \cdot x + 5)$$

$$(dy/dx)/y = -\ln(2 \cdot x + 5) + (-1 \cdot x + 2) \cdot \frac{2}{2 \cdot x + 5}$$

$$\text{Answer: } dy/dx = -3 \cdot (2 \cdot x + 5)^{-1 \cdot x + 2} \cdot \left(-\ln(2 \cdot x + 5) + (-1 \cdot x + 2) \cdot \frac{2}{2 \cdot x + 5} \right)$$

Original domain: $2 \cdot x + 5 > 0$. Logarithmic formula: $2 \cdot x + 5 > 0$.

Logarithmic Differentiation

Worked Answer Key

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

5. $y =$

$$\left(2 \cdot x^2 + 1\right)^{\frac{3}{2}} \cdot (-3 \cdot x + 4)^3$$

$$\ln|y| = \ln\left(\left|1\right|\right) + \frac{3}{2} \cdot \ln\left(\left|2 \cdot x^2 + 1\right|\right) + 3 \cdot \ln\left(\left|-3 \cdot x + 4\right|\right)$$

$$(dy/dx)/y = \frac{3}{2} \cdot \frac{4 \cdot x}{2 \cdot x^2 + 1} + 3 \cdot \frac{-3}{-3 \cdot x + 4}$$

$$\text{Answer: } dy/dx = \left(2 \cdot x^2 + 1\right)^{\frac{3}{2}} \cdot (-3 \cdot x + 4)^3 \cdot \left(\frac{3}{2} \cdot \frac{4 \cdot x}{2 \cdot x^2 + 1} + 3 \cdot \frac{-3}{-3 \cdot x + 4}\right)$$

Original domain: $2^*(x)^{(2)} + 1 > 0$. Logarithmic formula: $2^*(x)^{(2)} + 1 > 0$; $-3^*x + 4 \neq 0$.

At $x = 4/3$, f' exists and equals 0.

6. $y =$

$$\frac{-2 \cdot (-2 \cdot x + 4)^3 \cdot (2 \cdot x + 4)^3}{(3 \cdot x^2 + 1)^{\frac{1}{2}}}$$

$$\ln|y| = \ln\left(\left|-2\right|\right) + 3 \cdot \ln\left(\left|-2 \cdot x + 4\right|\right) + 3 \cdot \ln\left(\left|2 \cdot x + 4\right|\right) - \frac{1}{2} \cdot \ln\left(\left|3 \cdot x^2 + 1\right|\right)$$

$$(dy/dx)/y = 3 \cdot \frac{-2}{-2 \cdot x + 4} + 3 \cdot \frac{2}{2 \cdot x + 4} - \frac{1}{2} \cdot \frac{6 \cdot x}{3 \cdot x^2 + 1}$$

$$\text{Answer: } dy/dx = \frac{-2 \cdot (-2 \cdot x + 4)^3 \cdot (2 \cdot x + 4)^3}{(3 \cdot x^2 + 1)^{\frac{1}{2}}} \cdot \left(3 \cdot \frac{-2}{-2 \cdot x + 4} + 3 \cdot \frac{2}{2 \cdot x + 4} - \frac{1}{2} \cdot \frac{6 \cdot x}{3 \cdot x^2 + 1}\right)$$

Original domain: $3^*(x)^{(2)} + 1 > 0$. Logarithmic formula: $-2^*x + 4 \neq 0$; $2^*x + 4 \neq 0$; $3^*(x)^{(2)} + 1 > 0$.

At $x = 2$, f' exists and equals 0. At $x = -2$, f' exists and equals 0.

7. $y =$

$$2 \cdot (x^2 + 3)^{x+1}$$

$$\ln|y| = \ln\left(\left|2\right|\right) + (x - 1) \cdot \ln\left(\left|x^2 + 3\right|\right)$$

$$(dy/dx)/y = \ln\left(\left|x^2 + 3\right|\right) + (x - 1) \cdot \frac{2 \cdot x}{x^2 + 3}$$

$$\text{Answer: } dy/dx = 2 \cdot (x^2 + 3)^{x+1} \cdot \left(\ln\left(\left|x^2 + 3\right|\right) + (x - 1) \cdot \frac{2 \cdot x}{x^2 + 3}\right)$$

Original domain: $(x)^{(2)} + 3 > 0$. Logarithmic formula: $(x)^{(2)} + 3 > 0$.

Logarithmic Differentiation

Worked Answer Key

Differentiate each function using logarithmic differentiation. State the real domain used by the logarithmic derivative formula.

8. $y =$

$$-2 \cdot (2 \cdot x^2 + 2)^{\frac{3}{2}} \cdot (3 \cdot x + 2)^{\frac{1}{2}} \cdot (-2 \cdot x + 5)^3$$

$$\ln|y| = \ln\left(|-2|\right) + \frac{3}{2} \cdot \ln\left(|2 \cdot x^2 + 2|\right) + \frac{1}{2} \cdot \ln\left(|3 \cdot x + 2|\right) + 3 \cdot \ln\left(|-2 \cdot x + 5|\right)$$

$$(dy/dx)/y = \frac{3}{2} \cdot \frac{4 \cdot x}{2 \cdot x^2 + 2} + \frac{1}{2} \cdot \frac{3}{3 \cdot x + 2} + 3 \cdot \frac{-2}{-2 \cdot x + 5}$$

$$\text{Answer: } dy/dx = -2 \cdot (2 \cdot x^2 + 2)^{\frac{3}{2}} \cdot (3 \cdot x + 2)^{\frac{1}{2}} \cdot (-2 \cdot x + 5)^3 \cdot \left(\frac{3}{2} \cdot \frac{4 \cdot x}{2 \cdot x^2 + 2} + \frac{1}{2} \cdot \frac{3}{3 \cdot x + 2} + 3 \cdot \frac{-2}{-2 \cdot x + 5} \right)$$

Original domain: $2^*(x)^{(2)} + 2 >= 0$; $3*x + 2 >= 0$. Logarithmic formula: $2^*(x)^{(2)} + 2 > 0$; $3*x + 2 > 0$; $-2*x + 5 != 0$.

At $x = -2/3$, the point is a one-sided domain endpoint. At $x = 5/2$, f exists and equals 0.

9. $y =$

$$\frac{(2 \cdot x + 3)^{\frac{2}{3}}}{(-2 \cdot x + 5)^{\frac{1}{3}}}$$

$$\ln|y| = \ln\left(|1|\right) + \frac{2}{3} \cdot \ln\left(|2 \cdot x + 3|\right) - \frac{1}{3} \cdot \ln\left(|-2 \cdot x + 5|\right)$$

$$(dy/dx)/y = \frac{2}{3} \cdot \frac{2}{2 \cdot x + 3} - \frac{1}{3} \cdot \frac{-2}{-2 \cdot x + 5}$$

$$\text{Answer: } dy/dx = \frac{(2 \cdot x + 3)^{\frac{2}{3}}}{(-2 \cdot x + 5)^{\frac{1}{3}}} \cdot \left(\frac{2}{3} \cdot \frac{2}{2 \cdot x + 3} - \frac{1}{3} \cdot \frac{-2}{-2 \cdot x + 5} \right)$$

Original domain: $-2*x + 5 != 0$. Logarithmic formula: $2*x + 3 != 0$; $-2*x + 5 != 0$.

At $x = -3/2$, no finite two-sided derivative exists.

10. $y =$

$$-3 \cdot (x + 5)^{-3 \cdot x + 1}$$

$$\ln|y| = \ln\left(|-3|\right) + (-3 \cdot x - 1) \cdot \ln(x + 5)$$

$$(dy/dx)/y = -3 \cdot \ln(x + 5) + (-3 \cdot x - 1) \cdot \frac{1}{x + 5}$$

$$\text{Answer: } dy/dx = -3 \cdot (x + 5)^{-3 \cdot x + 1} \cdot \left(-3 \cdot \ln(x + 5) + (-3 \cdot x - 1) \cdot \frac{1}{x + 5} \right)$$

Original domain: $x + 5 > 0$. Logarithmic formula: $x + 5 > 0$.