

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Linear Approximation

Construct $L(x) = f(a) + f'(a)(x - a)$, then use $L(b)$ to estimate $f(b)$. Round only the final estimate to four decimal places.

1. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = -x^2 + 3$$

$$a = 0$$

$$b = \frac{-1}{20}$$

$$L(x) = \underline{\hspace{10cm}}$$

$$\text{Exact } L(b) = \underline{\hspace{10cm}}$$

$$f(b) \text{ approx. } \underline{\hspace{10cm}}$$

2. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = 2 \cdot e^x$$

$$a = -1$$

$$b = \frac{-19}{20}$$

$$L(x) = \underline{\hspace{10cm}}$$

$$\text{Exact } L(b) = \underline{\hspace{10cm}}$$

$$f(b) \text{ approx. } \underline{\hspace{10cm}}$$

3. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = \frac{1}{2} \cdot \sin\left(\frac{-1}{2} \cdot x\right) - 1$$

$$a = -1 \cdot \pi$$

$$b = -\pi + \frac{1}{20}$$

$$L(x) = \underline{\hspace{10cm}}$$

$$\text{Exact } L(b) = \underline{\hspace{10cm}}$$

$$f(b) \text{ approx. } \underline{\hspace{10cm}}$$

4. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = -3 \cdot x + x^3 - 3$$

$$a = 1$$

$$b = \frac{19}{20}$$

$$L(x) = \underline{\hspace{10cm}}$$

$$\text{Exact } L(b) = \underline{\hspace{10cm}}$$

$$f(b) \text{ approx. } \underline{\hspace{10cm}}$$

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Construct $L(x) = f(a) + f'(a)(x - a)$, then use $L(b)$ to estimate $f(b)$. Round only the final estimate to four decimal places.

5. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = -3 \cdot (2)^{-3 \cdot x - 2} + 1$$

$$a = -1$$

$$b = \frac{-21}{20}$$

$$L(x) = \underline{\hspace{10em}}$$

$$\text{Exact } L(b) = \underline{\hspace{10em}}$$

$$f(b) \text{ approx. } \underline{\hspace{10em}}$$

6. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = 2 \cdot \cos\left(\frac{1}{2} \cdot x\right) - 2$$

$$a = 0$$

$$b = \frac{-1}{50}$$

$$L(x) = \underline{\hspace{10em}}$$

$$\text{Exact } L(b) = \underline{\hspace{10em}}$$

$$f(b) \text{ approx. } \underline{\hspace{10em}}$$

7. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = 3 \cdot x - 2 \cdot x^2 + 3 \cdot x^3 + 2 \cdot x^4 - 1$$

$$a = 2$$

$$b = \frac{101}{50}$$

$$L(x) = \underline{\hspace{10em}}$$

$$\text{Exact } L(b) = \underline{\hspace{10em}}$$

$$f(b) \text{ approx. } \underline{\hspace{10em}}$$

8. Construct $L(x)$, then estimate $f(b)$.

$$f(x) = 3 \cdot (3)^{2 \cdot x - 1}$$

$$a = 1$$

$$b = \frac{21}{20}$$

$$L(x) = \underline{\hspace{10em}}$$

$$\text{Exact } L(b) = \underline{\hspace{10em}}$$

$$f(b) \text{ approx. } \underline{\hspace{10em}}$$

Linear Approximation

Worked Answer Key

Construct $L(x) = f(a) + f'(a)(x - a)$, then use $L(b)$ to estimate $f(b)$. Round only the final estimate to four decimal places.

1. $f(x) = -x^2 + 3$
 $f'(x) = -2 \cdot x$
 $a = 0$
 $b = \frac{-1}{20}$
 $f(a) = 3$
 $f'(a) = 0$
 $L(x) = 3$
 Exact $L(b) = 3$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately 3.0000 after rounding to four decimal places.
 All real x .

2. $f(x) = 2 \cdot e^x$
 $f'(x) = 2 \cdot e^x$
 $a = -1$
 $b = \frac{-19}{20}$
 $f(a) = 2 \cdot e^{-1}$
 $f'(a) = 2 \cdot e^{-1}$
 $L(x) = 2 \cdot e^{-1} + 2 \cdot e^{-1} \cdot (x + 1)$
 Exact $L(b) = 2 \cdot e^{-1} + \frac{1}{10} \cdot e^{-1}$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately 0.7725 after rounding to four decimal places.
 All real x .

3. $f(x) = \frac{1}{2} \cdot \sin\left(\frac{-1}{2} \cdot x\right) - 1$
 $f'(x) = \frac{-1}{4} \cdot \cos\left(\frac{-1}{2} \cdot x\right)$
 $a = -1 \cdot \pi$
 $b = -\pi + \frac{1}{20}$
 $f(a) = \frac{-1}{2}$
 $f'(a) = 0$
 $L(x) = \frac{-1}{2}$
 Exact $L(b) = \frac{-1}{2}$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately -0.5000 after rounding to four decimal places.
 All real x . Angles are in radians; a and b stay in the same domain component.

Linear Approximation

Worked Answer Key

Construct $L(x) = f(a) + f'(a)(x - a)$, then use $L(b)$ to estimate $f(b)$. Round only the final estimate to four decimal places.

4. $f(x) = -3 \cdot x + x^3 - 3$

$$f'(x) = 3 \cdot x^2 - 3$$

$$a = 1$$

$$b = \frac{19}{20}$$

$$f(a) = -5$$

$$f'(a) = 0$$

$$L(x) = -5$$

$$\text{Exact } L(b) = -5$$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately -5.0000 after rounding to four decimal places.

All real x .

5. $f(x) = -3 \cdot (2)^{-3 \cdot x - 2} + 1$

$$f'(x) = 9 \cdot \ln(2) \cdot (2)^{-3 \cdot x - 2}$$

$$a = -1$$

$$b = \frac{-21}{20}$$

$$f(a) = -5$$

$$f'(a) = 18 \cdot \ln(2)$$

$$L(x) = 18 \cdot \ln(2) \cdot (x + 1) - 5$$

$$\text{Exact } L(b) = -\frac{9}{10} \cdot \ln(2) - 5$$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately -5.6238 after rounding to four decimal places.

All real x .

6. $f(x) = 2 \cdot \cos\left(\frac{1}{2} \cdot x\right) - 2$

$$f'(x) = -1 \cdot \sin\left(\frac{1}{2} \cdot x\right)$$

$$a = 0$$

$$b = \frac{-1}{50}$$

$$f(a) = 0$$

$$f'(a) = 0$$

$$L(x) = 0$$

$$\text{Exact } L(b) = 0$$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately 0.0000 after rounding to four decimal places.

All real x . Angles are in radians; a and b stay in the same domain component.

Linear Approximation

Worked Answer Key

Construct $L(x) = f(a) + f'(a)(x - a)$, then use $L(b)$ to estimate $f(b)$. Round only the final estimate to four decimal places.

7. $f(x) = 3 \cdot x - 2 \cdot x^2 + 3 \cdot x^3 + 2 \cdot x^4 - 1$

$$f'(x) = -4 \cdot x + 9 \cdot x^2 + 8 \cdot x^3 + 3$$

$$a = 2$$

$$b = \frac{101}{50}$$

$$f(a) = 53$$

$$f'(a) = 95$$

$$L(x) = 95 \cdot (x - 2) + 53$$

$$\text{Exact } L(b) = \frac{549}{10}$$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately 54.9000 after rounding to four decimal places.

All real x .

8. $f(x) = 3 \cdot (3)^{2 \cdot x - 1}$

$$f'(x) = 6 \cdot \ln(3) \cdot (3)^{2 \cdot x - 1}$$

$$a = 1$$

$$b = \frac{21}{20}$$

$$f(a) = 9$$

$$f'(a) = 18 \cdot \ln(3)$$

$$L(x) = 18 \cdot \ln(3) \cdot (x - 1) + 9$$

$$\text{Exact } L(b) = \frac{9}{10} \cdot \ln(3) + 9$$

Since $L(b)$ is the exact linear-model value, $f(b)$ is approximately 9.9888 after rounding to four decimal places.

All real x .