

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Evaluating Limits at Infinity

Evaluate each limit exactly. Show which leading term, bounded factor, or growth comparison controls the tail.

1.

$$\lim_{x \rightarrow \infty} 8 \cdot x^3 + 9 \cdot x^2 - 2 \cdot x + 7$$

Exact limit: _____

2.

$$\lim_{x \rightarrow +\infty} \frac{3 \cdot x^2 + x + 5}{9 \cdot x^2 - 2 \cdot x - 8}$$

Exact limit: _____

3.

$$\lim_{x \rightarrow \infty} 1 - 3 \cdot \tan\left(-x + 2\right)$$

Exact limit: _____

4.

$$\lim_{x \rightarrow +\infty} -3 + 3 \cdot \left(3\right)^{-x - 1}$$

Exact limit: _____

5.

$$\lim_{x \rightarrow +\infty} -8 \cdot x^4 - 5 \cdot x^3 - 2 \cdot x^2 - 7 \cdot x + 1$$

Exact limit: _____

6.

$$\lim_{x \rightarrow \infty} \frac{-8 \cdot x^4 + 9 \cdot x^3 - 6 \cdot x^2 - 9 \cdot x - 9}{-8 \cdot x - 4}$$

Exact limit: _____

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7.

$$\lim_{x \rightarrow +\infty} 3 + \frac{1}{2} \cdot \frac{\sin(2 \cdot x + 3)}{(x - 1)^2}$$

Exact limit: _____

8.

$$\lim_{x \rightarrow -\infty} 3 - 3 \cdot \frac{x^4}{(2)^{2 \cdot x + 1}}$$

Exact limit: _____

Evaluating Limits at Infinity

Worked Answer Key

Evaluate each limit exactly. Show which leading term, bounded factor, or growth comparison controls the tail.

1. Evaluate the tail exactly.

$$\lim_{x \rightarrow -\infty} 8 \cdot x^3 + 9 \cdot x^2 - 2 \cdot x + 7$$

Degree 3 and leading coefficient 8 control the tail; the direction contributes sign -1.

$$\text{Limit} = -\infty$$

2. Evaluate the tail exactly.

$$\lim_{x \rightarrow +\infty} \frac{3 \cdot x^2 + x + 5}{9 \cdot x^2 - 2 \cdot x - 8}$$

Compare degrees 2 and 2. Their difference is 0, with leading-coefficient ratio 1/3.

$$\text{Limit} = \frac{1}{3}$$

Evaluating Limits at Infinity

Worked Answer Key

Evaluate each limit exactly. Show which leading term, bounded factor, or growth comparison controls the tail.

3. Evaluate the tail exactly.

$$\lim_{x \rightarrow -\infty} 1 - 3 \cdot \tan(-x + 2)$$

$\tan$ keeps oscillating on this tail, so it has no single extended-real limit.

Limit = does not exist

4. Evaluate the tail exactly.

$$\lim_{x \rightarrow +\infty} -3 + 3 \cdot (3)^{-x - 1}$$

The signed exponent rate makes the exponential tend to 0 on this tail.

Limit = -3

Evaluating Limits at Infinity

Worked Answer Key

Evaluate each limit exactly. Show which leading term, bounded factor, or growth comparison controls the tail.

5. Evaluate the tail exactly.

$$\lim_{x \rightarrow +\infty} -8x^4 - 5x^3 - 2x^2 - 7x + 1$$

Degree 4 and leading coefficient -8 control the tail; the direction contributes sign 1.

$$\text{Limit} = -\infty$$

6. Evaluate the tail exactly.

$$\lim_{x \rightarrow -\infty} \frac{-8x^4 + 9x^3 - 6x^2 - 9x - 9}{-8x - 4}$$

Compare degrees 4 and 1. Their difference is 3, with leading-coefficient ratio 1.

$$\text{Limit} = -\infty$$

Evaluating Limits at Infinity

Worked Answer Key

Evaluate each limit exactly. Show which leading term, bounded factor, or growth comparison controls the tail.

7. Evaluate the tail exactly.

$$\lim_{x \rightarrow +\infty} 3 + \frac{1}{2} \cdot \frac{\sin(2 \cdot x + 3)}{(x - 1)^2}$$

sin stays bounded while the power-2 denominator grows in magnitude, so the quotient tends to 0.

Limit = 3

8. Evaluate the tail exactly.

$$\lim_{x \rightarrow -\infty} 3 - 3 \cdot \frac{x^4}{(2)^{-2 \cdot x + 1}}$$

Exponential growth dominates every fixed polynomial power; track the sign of x^4 .

Limit = -3