

Limits at Essential Discontinuities

Evaluate each requested finite-point limit exactly.

1. $f(x) = \frac{2}{(x+2)(x+2)(x+2)}$

$\lim_{x \rightarrow -2^+} f(x) = \underline{\hspace{2cm}}$

2. $f(x) = 2 - \cot(-1 \cdot (x+2))$

$\lim_{x \rightarrow -2^-} f(x) = \underline{\hspace{2cm}}$

3. $f(x) = \frac{\frac{1}{2}}{(x-1)}$

$\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$

4. $f(x) = \frac{-1 \cdot x \cdot (x-1)(x-2)}{(x+1)}$

$\lim_{x \rightarrow -1^-} f(x) = \underline{\hspace{2cm}}$

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Evaluate each requested finite-point limit exactly.

5. $f(x) =$

$$-2 - 2 \cdot \csc\left(\frac{1}{2} \cdot (x+1)\right)$$

$\lim_{x \rightarrow -1^+} f(x) =$ _____

6. $f(x) =$

$$\frac{\frac{1}{2}}{(x+2)(x+2)}$$

$\lim_{x \rightarrow -2^+} f(x) =$ _____

7. $f(x) =$

$$\frac{-2 \cdot (x+1) \cdot (x-1)}{(x-2)(x-2)(x-2)}$$

$\lim_{x \rightarrow 2^+} f(x) =$ _____

8. $f(x) =$

$$0 - 2 \cdot \tan\left(\frac{1}{2} \cdot \pi + (x+2)\right)$$

$\lim_{x \rightarrow -2^-} f(x) =$ _____

Limits at Essential Discontinuities

Worked Answer Key

Evaluate each requested finite-point limit exactly.

1. $f(x) = \frac{2}{(x+2)(x+2)(x+2)}$

$\lim_{x \rightarrow -2^+} f(x) = +\infty$

Residual pole order 3; leading coefficient 2. Residual denominator multiplicity and the exact leading sign determine the one-sided unbounded behavior. Original domain: Original denominator is nonzero.

2. $f(x) = 2 - \cot(-1 \cdot (x+2))$

$\lim_{x \rightarrow -2^-} f(x) = -\infty$

$\cot(u) = \cos(u)/\sin(u)$, $\csc(u) = 1/\sin(u)$ Pole order 1; leading coefficient 1. The reciprocal has opposite exact signs on the two sides of the pole. Original domain: The original sine or cosine denominator must be nonzero.

Limits at Essential Discontinuities

Worked Answer Key

Evaluate each requested finite-point limit exactly.

3. $f(x) = \frac{\frac{1}{2}}{(x-1)}$

$\lim_{x \rightarrow 1^-} f(x) = -\text{infinity}$

Residual pole order 1; leading coefficient 1/2. Residual denominator multiplicity and the exact leading sign determine the one-sided unbounded behavior. Original domain: Original denominator is nonzero.

4. $f(x) = \frac{-1 \cdot x \cdot (x-1) \cdot (x-2)}{(x+1)}$

$\lim_{x \rightarrow -1^-} f(x) = -\text{infinity}$

Residual pole order 1; leading coefficient 6. Residual denominator multiplicity and the exact leading sign determine the one-sided unbounded behavior. Original domain: Original denominator is nonzero.

Limits at Essential Discontinuities

Worked Answer Key

Evaluate each requested finite-point limit exactly.

5. $f(x) = -2 - 2 \cdot \csc\left(\frac{1}{2} \cdot (x + 1)\right)$

$\lim_{x \rightarrow -1^+} f(x) = -\infty$

$\cot(u) = \cos(u)/\sin(u)$, $\csc(u) = 1/\sin(u)$ Pole order 1; leading coefficient -4. The reciprocal has opposite exact signs on the two sides of the pole. Original domain: The original sine or cosine denominator must be nonzero.

6. $f(x) = \frac{1}{(x+2)(x+2)}$

$\lim_{x \rightarrow -2^+} f(x) = +\infty$

Residual pole order 2; leading coefficient 1/2. Residual denominator multiplicity and the exact leading sign determine the one-sided unbounded behavior. Original domain: Original denominator is nonzero.

Limits at Essential Discontinuities

Worked Answer Key

Evaluate each requested finite-point limit exactly.

7. $f(x) =$

$$\frac{-2 \cdot (x+1)(x-1)}{(x-2)(x-2)(x-2)}$$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

Residual pole order 3; leading coefficient -6. Residual denominator multiplicity and the exact leading sign determine the one-sided unbounded behavior. Original domain: Original denominator is nonzero.

8. $f(x) =$

$$0 - 2 \cdot \tan\left(\frac{1}{2} \cdot \pi + (x+2)\right)$$

$$\lim_{x \rightarrow -2^-} f(x) = -\infty$$

$\tan(\pi/2+u)=-\cot(u)$, $\sec(\pi/2+u)=-\csc(u)$ Pole order 1; leading coefficient 2. The reciprocal has opposite exact signs on the two sides of the pole. Original domain: The original sine or cosine denominator must be nonzero.