Name _____ Score _____ Teacher _____ Date _____

L'Hopital's Rule

Confirm the indeterminate form, differentiate numerator and denominator separately, and continue only while L'Hopital's Rule applies. Give an exact limit, signed infinity, or DNE.

1. exponential logarithmic

$$\lim_{x \to 2} \frac{e^{x-2} - 1}{-5 \cdot (x-2)^2}$$

Indeterminate form: _____

_____

Exact limit: _____

2. polynomial trigonometric

$$\lim_{x \to 2^-} \frac{6 \cdot \sin(4 \cdot (x-2))}{6 \cdot (x-2)^2}$$

Indeterminate form: _____

_____

Exact limit: _____

3. polynomial

$$\lim_{x \to 4} \frac{336 \cdot 1 - 228 \cdot x + 52 \cdot x^2 - 4 \cdot x^3}{-32 \cdot 1 + 16 \cdot x - 2 \cdot x^2}$$

Indeterminate form: _____

_____

Exact limit: _____

4. trigonometric

$$\lim_{x \to -4} \frac{-3 \cdot \sin(x+4)}{3 \cdot (1 - \cos(2 \cdot (x+4)))}$$

Indeterminate form: _____

_____

Exact limit: _____

Name _____ Score _____ Teacher _____ Date _____

L'Hopital's Rule

Confirm the indeterminate form, differentiate numerator and denominator separately, and continue only while L'Hopital's Rule applies. Give an exact limit, signed infinity, or DNE.

5. polynomial

$$\lim_{x \to +\infty} \frac{-x + 3 \cdot x^2}{-2 \cdot 1 + x + 3 \cdot x^3 - 5 \cdot x^4}$$

Indeterminate form: _____

_____

Exact limit: _____

6. trigonometric

$$\lim_{x \to 0^-} \frac{4 \cdot \tan(4 \cdot x)}{\sin(4 \cdot x)}$$

Indeterminate form: _____

_____

Exact limit: _____

7. exponential logarithmic

$$\lim_{x \to 0^-} \frac{-6 \cdot x^3}{-4 \cdot \left(e^{-3 \cdot x} - 1 + 3 \cdot x\right)}$$

Indeterminate form: _____

_____

Exact limit: _____

8. polynomial trigonometric

$$\lim_{x \to -3^+} \frac{-2 \cdot (x+3)^3}{-4 \cdot \tan(x+3)}$$

Indeterminate form: _____

_____

Exact limit: _____

Name _____ Score _____ Teacher _____ Date _____

L'Hopital's Rule

Worked Answer Key

Confirm the indeterminate form, differentiate numerator and denominator separately, and continue only while L'Hopital's Rule applies. Give an exact limit, signed infinity, or DNE.

1. 0/0 form

$$\lim_{x \to 2} \frac{e^{x-2} - 1}{-5 \cdot (x-2)^2}$$

Applicability: Both functions are differentiable and every denominator derivative used is nonzero on this punctured neighborhood.

Rule 1 (0/0):

$$\lim_{x \to 2} \frac{e^{x-2}}{-10 \cdot (x-2)}$$

After 1 justified application, the exact lowest-order terms determine the terminal limit.

Limit = DNE (left $+\infty$, right $-\infty$)

The original denominator is nonzero in the certified punctured neighborhood.

2. 0/0 form

$$\lim_{x \to 2^-} \frac{6 \cdot \sin(4 \cdot (x-2))}{6 \cdot (x-2)^2}$$

Applicability: Both functions are differentiable and every denominator derivative used is nonzero on this punctured neighborhood.

Rule 1 (0/0):

$$\lim_{x \to 2^-} \frac{24 \cdot \cos(4 \cdot (x-2))}{12 \cdot (x-2)}$$

After 1 justified application, the exact lowest-order terms determine the terminal limit.

Limit = $-\infty$

The original denominator is nonzero away from its isolated zeros in the certified punctured neighborhood.

Name _____ Score _____ Teacher _____ Date _____

L'Hopital's Rule

Worked Answer Key

Confirm the indeterminate form, differentiate numerator and denominator separately, and continue only while L'Hopital's Rule applies. Give an exact limit, signed infinity, or DNE.

3. 0/0 form

$$\lim_{x \to 4} \frac{336 \cdot 1 - 228 \cdot x + 52 \cdot x^2 - 4 \cdot x^3}{-32 \cdot 1 + 16 \cdot x - 2 \cdot x^2}$$

Applicability: After factoring the original denominator and every denominator derivative used into its first nonzero power of x-a, each remaining factor stays nonzero throughout this punctured neighborhood.

Rule 1 (0/0):

$$\lim_{x \to 4} \frac{-228 \cdot 1 + 104 \cdot x - 12 \cdot x^2}{16 \cdot 1 - 4 \cdot x}$$

After 1 separate derivative application, compare the first nonzero orders exactly.

Limit = DNE (left $-\infty$, right $+\infty$)

The original denominator is nonzero except at its isolated roots; x equals the approach point is excluded.

4. 0/0 form

$$\lim_{x \to -4} \frac{-3 \cdot \sin(x+4)}{3 \cdot (1 - \cos(2 \cdot (x+4)))}$$

Applicability: The trigonometric functions are differentiable and the denominator derivative used is nonzero on this punctured neighborhood.

Rule 1 (0/0):

$$\lim_{x \to -4} \frac{-3 \cdot \cos(x+4)}{6 \cdot \sin(2 \cdot (x+4))}$$

The exact trigonometric derivative chain resolves the limit after 1 application.

Limit = DNE (left $+\infty$, right $-\infty$)

The original trigonometric denominator must be nonzero; the approach point itself is excluded.

Name _____ Score _____ Teacher _____ Date _____

L'Hopital's Rule

Worked Answer Key

Confirm the indeterminate form, differentiate numerator and denominator separately, and continue only while L'Hopital's Rule applies. Give an exact limit, signed infinity, or DNE.

5. infinity/infinity form

$$\lim_{x \to +\infty} \frac{-x + 3 \cdot x^2}{-2 \cdot 1 + x + 3 \cdot x^3 - 5 \cdot x^4}$$

Applicability: Beyond this Cauchy bound, the original denominator and every denominator derivative used have the sign of their leading terms and are nonzero.

Rule 1 (infinity/infinity):

$$\lim_{x \to +\infty} \frac{-1 + 6 \cdot x}{1 + 9 \cdot x^2 - 20 \cdot x^3}$$

Rule 2 (still infinity/infinity):

$$\lim_{x \to +\infty} \frac{6 \cdot 1}{18 \cdot x - 60 \cdot x^2}$$

Differentiate separately 2 times, then compare the remaining degree and leading sign.

Limit = 0

The original denominator is nonzero on the certified tail.

6. 0/0 form

$$\lim_{x \to 0^-} \frac{4 \cdot \tan(4 \cdot x)}{\sin(4 \cdot x)}$$

Applicability: The trigonometric functions are differentiable and the denominator derivative used is nonzero on this punctured neighborhood.

Rule 1 (0/0):

$$\lim_{x \to 0^-} \frac{16 \cdot \sec(4 \cdot x)^2}{4 \cdot \cos(4 \cdot x)}$$

The exact trigonometric derivative chain resolves the limit after 1 application.

Limit = 4

The original trigonometric denominator must be nonzero; the approach point itself is excluded.

Name _____ Score _____ Teacher _____ Date _____

L'Hopital's Rule

Worked Answer Key

Confirm the indeterminate form, differentiate numerator and denominator separately, and continue only while L'Hopital's Rule applies. Give an exact limit, signed infinity, or DNE.

7. 0/0 form

$$\lim_{x \to 0^-} \frac{-6 \cdot x^3}{-4 \cdot \left(e^{-3 \cdot x} - 1 + 3 \cdot x\right)}$$

Applicability: Both functions are differentiable and every denominator derivative used is nonzero on this punctured neighborhood.

Rule 1 (0/0):

$$\lim_{x \to 0^-} \frac{-18 \cdot x^2}{12 \cdot \left(e^{-3 \cdot x} - 1\right)}$$

Rule 2 (still 0/0):

$$\lim_{x \to 0^-} \frac{-36 \cdot x}{-36 \cdot e^{-3 \cdot x}}$$

After 2 justified applications, the exact lowest-order terms determine the terminal limit.

Limit = 0

The original denominator is nonzero in the certified punctured neighborhood.

8. 0/0 form

$$\lim_{x \to -3^+} \frac{-2 \cdot (x+3)^3}{-4 \cdot \tan(x+3)}$$

Applicability: Both functions are differentiable and every denominator derivative used is nonzero on this punctured neighborhood.

Rule 1 (0/0):

$$\lim_{x \to -3^+} \frac{-6 \cdot (x+3)^2}{-4 \cdot \sec(x+3)^2}$$

After 1 justified application, the exact lowest-order terms determine the terminal limit.

Limit = 0

The original denominator is nonzero away from its isolated zeros in the certified punctured neighborhood.