

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Inverse Trigonometric Integration

Use the requested inverse trigonometric function, adjust for its scaled affine derivative pattern, and include + C. Mathematically equivalent antiderivatives are accepted.

1. Use arccos:

$$\int 2 \cdot \frac{1}{\sqrt{1 - (x)^2}} dx$$

6. Use arccsc:

$$\int \frac{-2}{3} \cdot \frac{1}{|x| \sqrt{(x)^2 - 1}} dx$$

2. Use arctan:

$$\int \frac{-3}{2} \cdot \frac{1}{1 + (x)^2} dx$$

7. Use arccot:

$$\int \frac{-2}{3} \cdot \frac{1}{4 + (-2 \cdot x)^2} dx$$

3. Use arccot:

$$\int \frac{1}{1 + (x)^2} dx$$

8. Use arccos:

$$\int 3 \cdot \frac{1}{\sqrt{16 - (-2 - 3 \cdot x)^2}} dx$$

4. Use arcsin:

$$\int \frac{3}{2} \cdot \frac{1}{\sqrt{1 - (x)^2}} dx$$

9. Use arctan:

$$\int \frac{1}{2} \cdot \frac{1}{9 + (-3 \cdot x)^2} dx$$

5. Use arcsec:

$$\int 2 \cdot \frac{1}{|x| \sqrt{(x)^2 - 1}} dx$$

10. Use arccsc:

$$\int 3 \cdot \frac{1}{|1 + 3 \cdot x| \sqrt{(1 + 3 \cdot x)^2 - 16}} dx$$

Inverse Trigonometric Integration

Worked Answer Key

Use the requested inverse trigonometric function, adjust for its scaled affine derivative pattern, and include + C. Mathematically equivalent antiderivatives are accepted.

1. Original: $\int 2 \cdot \frac{1}{\sqrt{1-(x)^2}} dx$

Pattern: $d/dx \arccos\left(\frac{x}{1}\right) = -1 \cdot \frac{1}{\sqrt{1-(x)^2}}$

Coefficient: $\frac{2}{-1} = -2$

Result: $-2 \cdot \arccos\left(\frac{x}{1}\right) + C$

Original domain: $|x| < 1$

On each open domain component; constants may differ on disconnected components.

2. Original: $\int \frac{-3}{2} \cdot \frac{1}{1+(x)^2} dx$

Pattern: $d/dx \arctan\left(\frac{x}{1}\right) = \frac{1}{1+(x)^2}$

Coefficient: $\frac{-3}{2} = \frac{-3}{2}$

Result: $-\frac{3}{2} \cdot \arctan\left(\frac{x}{1}\right) + C$

Original domain: all real x.

On each open domain component; constants may differ on disconnected components.

3. Original: $\int \frac{1}{1+(x)^2} dx$

Pattern: $d/dx \operatorname{arccot}\left(\frac{x}{1}\right) = -1 \cdot \frac{1}{1+(x)^2}$

Coefficient: $\frac{1}{-1} = -1$

Result: $-\operatorname{arccot}\left(\frac{x}{1}\right) + C$

Original domain: all real x.

On each open domain component; constants may differ on disconnected components.

4. Original: $\int \frac{3}{2} \cdot \frac{1}{\sqrt{1-(x)^2}} dx$

Pattern: $d/dx \arcsin\left(\frac{x}{1}\right) = \frac{1}{\sqrt{1-(x)^2}}$

Coefficient: $\frac{3}{2} = \frac{3}{2}$

Result: $\frac{3}{2} \cdot \arcsin\left(\frac{x}{1}\right) + C$

Original domain: $|x| < 1$

On each open domain component; constants may differ on disconnected components.

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5. Original: $\int 2 \cdot \frac{1}{|x| \sqrt{(x)^2 - 1}} dx$

Pattern: $d/dx \quad \operatorname{arcsec}\left(\frac{x}{1}\right) = \frac{1}{|x| \sqrt{(x)^2 - 1}}$

Coefficient: $\frac{2}{1} = 2$

Result: $2 \cdot \operatorname{arcsec}\left(\frac{x}{1}\right) + C$

Original domain: $|x| > 1$

On each open domain component; constants may differ on disconnected components.

6. Original: $\int \frac{-2}{3} \cdot \frac{1}{|x| \sqrt{(x)^2 - 1}} dx$

Pattern: $d/dx \quad \operatorname{arccsc}\left(\frac{x}{1}\right) = -1 \cdot \frac{1}{|x| \sqrt{(x)^2 - 1}}$

Coefficient: $\frac{\frac{-2}{3}}{-1} = \frac{2}{3}$

Result: $\frac{2}{3} \cdot \operatorname{arccsc}\left(\frac{x}{1}\right) + C$

Original domain: $|x| > 1$

On each open domain component; constants may differ on disconnected components.

7. Original: $\int \frac{-2}{3} \cdot \frac{1}{4 + (-2 \cdot x)^2} dx$

Pattern: $d/dx \quad \operatorname{arccot}\left(\frac{-2 \cdot x}{2}\right) = 4 \cdot \frac{1}{4 + (-2 \cdot x)^2}$

Coefficient: $\frac{\frac{-2}{3}}{4} = \frac{-1}{6}$

Result: $-\frac{1}{6} \cdot \operatorname{arccot}\left(\frac{-2 \cdot x}{2}\right) + C$

Original domain: all real x.

On each open domain component; constants may differ on disconnected components.

8. Original: $\int 3 \cdot \frac{1}{\sqrt{16 - (-2 - 3 \cdot x)^2}} dx$

Pattern: $d/dx \quad \arccos\left(\frac{-2 - 3 \cdot x}{4}\right) = 3 \cdot \frac{1}{\sqrt{16 - (-2 - 3 \cdot x)^2}}$

Coefficient: $\frac{3}{3} = 1$

Result: $\arccos\left(\frac{-2 - 3 \cdot x}{4}\right) + C$

Original domain: $|-2 - 3 \cdot x| < 4$

On each open domain component; constants may differ on disconnected components.

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9. Original: $\int \frac{1}{2} \cdot \frac{1}{9 + (-3 \cdot x)^2} dx$

Pattern: $d/dx \arctan\left(\frac{-3 \cdot x}{3}\right) = -9 \cdot \frac{1}{9 + (-3 \cdot x)^2}$

Coefficient: $\frac{\frac{1}{2}}{-9} = \frac{-1}{18}$

Result: $-\frac{1}{18} \cdot \arctan\left(\frac{-3 \cdot x}{3}\right) + C$

Original domain: all real x.

On each open domain component; constants may differ on disconnected components.

10. Original: $\int 3 \cdot \frac{1}{|1 + 3 \cdot x| \cdot \sqrt{(1 + 3 \cdot x)^2 - 16}} dx$

Pattern: $d/dx \operatorname{arccsc}\left(\frac{1 + 3 \cdot x}{4}\right) = -12 \cdot \frac{1}{|1 + 3 \cdot x| \cdot \sqrt{(1 + 3 \cdot x)^2 - 16}}$

Coefficient: $\frac{3}{-12} = \frac{-1}{4}$

Result: $-\frac{1}{4} \cdot \operatorname{arccsc}\left(\frac{1 + 3 \cdot x}{4}\right) + C$

Original domain: $|1 + 3 \cdot x| > 4$

On each open domain component; constants may differ on disconnected components.