

Derivatives of Inverse Functions

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

1. Find g and g'.

$f(x) = \frac{2 \cdot x - 7}{4}$

Original domain: all real numbers.

2. Find g and g'.

$f(x) = 2 \cdot (x - 1)^2 + 4$

Original domain:  $x \geq 1$ .

3. Find g and g'.

$f(x) = \frac{-3 \cdot x - 3}{x - 4}$

Original domain:  $x \neq 4$ .

4. Find g and g'.

$f(x) = \log_5(3 \cdot x - 2)$

Original domain:  $x > 2/3$ .

5. Find g and g'.

$f(x) = \frac{10^x + 4}{-4}$

Original domain: all real numbers.

6. Find g and g'.

$f(x) = \frac{5 \cdot x - 9}{3}$

Original domain: all real numbers.

7. Find g and g'.

$f(x) = -2 \cdot (x + 3)^3 - 1$

Original domain: all real numbers.

8. Find g and g'.

$f(x) = \frac{-1 \cdot x + 1}{3 \cdot x + 3}$

Original domain:  $x \neq -1$ .

9. Find g and g'.

$f(x) = \log_5(4 \cdot x - 4)$

Original domain:  $x > 1$ .

10. Find g and g'.

$f(x) = \frac{10^x - 3}{5}$

Original domain: all real numbers.

11. Find g and g'.

$f(x) = \frac{4 \cdot x - 1}{3}$

Original domain: all real numbers.

12. Find g and g'.

$f(x) = 3 \cdot (x + 4)^4 + 1$

Original domain:  $x \leq -4$ .

13. Find g and g'.

$f(x) = \frac{-2}{x - 3}$

Original domain:  $x \neq 3$ .

14. Find g and g'.

$f(x) = \log_3(5 \cdot x + 5)$

Original domain:  $x > -1$ .

15. Find g and g'.

$f(x) = \frac{5^x - 2}{-2}$

Original domain: all real numbers.

Find g first, then differentiate g directly. f inverse is a function, not 1/f.  
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Derivatives of Inverse Functions

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

16. Find  $g$  and  $g'$ .

$f(x) = \frac{6 \cdot x + 8}{5}$

Original domain: all real numbers.

17. Find  $g$  and  $g'$ .

$f(x) = -1 \cdot (x + 2)^2 - 1$

Original domain:  $x \leq -2$ .

18. Find  $g$  and  $g'$ .

$f(x) = \frac{4 \cdot x - 3}{2 \cdot x}$

Original domain:  $x \neq 0$ .

19. Find  $g$  and  $g'$ .

$f(x) = \log_{10}(-2 \cdot x - 2)$

Original domain:  $x < -1$ .

20. Find  $g$  and  $g'$ .

$f(x) = \frac{2^x - 4}{5}$

Original domain: all real numbers.

21. Find  $g$  and  $g'$ .

$f(x) = \frac{6 \cdot x - 8}{3}$

Original domain: all real numbers.

22. Find  $g$  and  $g'$ .

$f(x) = (x - 1)^3$

Original domain: all real numbers.

23. Find  $g$  and  $g'$ .

$f(x) = \frac{-2 \cdot x + 4}{x + 4}$

Original domain:  $x \neq -4$ .

24. Find  $g$  and  $g'$ .

$f(x) = \log_{10}(3 \cdot x - 5)$

Original domain:  $x > 5/3$ .

25. Find  $g$  and  $g'$ .

$f(x) = \frac{10^x + 4}{-2}$

Original domain: all real numbers.

26. Find  $g$  and  $g'$ .

$f(x) = \frac{3 \cdot x + 4}{3}$

Original domain: all real numbers.

27. Find  $g$  and  $g'$ .

$f(x) = 3 \cdot (x - 3)^4$

Original domain:  $x \geq 3$ .

28. Find  $g$  and  $g'$ .

$f(x) = \frac{x + 5}{3 \cdot x + 1}$

Original domain:  $x \neq -1/3$ .

29. Find  $g$  and  $g'$ .

$f(x) = \log_5(5 \cdot x - 4)$

Original domain:  $x > 4/5$ .

30. Find  $g$  and  $g'$ .

$f(x) = \frac{5^x - 3}{-2}$

Original domain: all real numbers.

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f^{-1}$ , then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

#### 1. $f(x)$

$$= \frac{2 \cdot x - 7}{4}$$

Swap x and y:

$$x = \frac{2 \cdot y - 7}{4}$$

Multiply by the denominator:

$$4 \cdot x = 2 \cdot y - 7$$

Isolate y:

$$y = \frac{4 \cdot x + 7}{2}$$

$$g(x) = \frac{4 \cdot x + 7}{2}$$

Differentiate g directly:

$$g' = 2$$

Original: all real numbers. Inverse: all real numbers. Derivative: all real numbers.

#### 2. $f(x)$

$$= 2 \cdot (x - 1)^2 + 4$$

Swap x and y:

$$x = 2 \cdot (y - 1)^2 + 4$$

Subtract k and divide by a:

$$(y - 1)^2 = \frac{x - 4}{2}$$

Take the selected real root:

$$y - 1 = \sqrt{\frac{x - 4}{2}}$$

Solve for y:

$$y = \sqrt{\frac{x - 4}{2}} + 1$$

$$g(x) = \sqrt{\frac{x - 4}{2}} + 1$$

Differentiate the explicit inverse:

$$g' = \frac{1}{4} \cdot \left( \frac{x - 4}{2} \right)^{-\frac{1}{2}}$$

Original:  $x \geq 4$ . Inverse:  $x \geq 4$ . Derivative:  $x > 4$ .

The inverse exists at  $x = 4$ , but its derivative is not finite there.

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

#### 3. $f(x)$

$$= \frac{-3 \cdot x - 3}{x - 4}$$

Swap x and y:

$$x = \frac{-3 \cdot y - 3}{y - 4}$$

Cross-multiply:

$$x \cdot (y - 4) = -3 \cdot y - 3$$

Collect the y terms:

$$(x + 3) \cdot y = -3 + 4 \cdot x$$

Solve for y:

$$y = \frac{-3 + 4 \cdot x}{x + 3}$$

$$g(x) = \frac{-3 + 4 \cdot x}{x + 3}$$

Apply the quotient rule:

$$g' = \frac{4 \cdot (x + 3) - 1 \cdot (-3 + 4 \cdot x)}{(x + 3)^2}$$

Collapse the numerator to the determinant:

$$g' = \frac{15}{(x + 3)^2}$$

Original:  $x \neq -4$ . Inverse:  $x \neq -3$ . Derivative:  $x \neq -3$ .

#### 4. $f(x)$

$$= \log_5(3 \cdot x - 2)$$

Swap x and y:

$$x = \log_5(3 \cdot y - 2)$$

Exponentiate:

$$5^x = 3 \cdot y - 2$$

Isolate y:

$$y = \frac{5^x + 2}{3}$$

$$g(x) = \frac{5^x + 2}{3}$$

Differentiate the exponential inverse:

$$g' = \frac{1}{3} \cdot \ln 5 \cdot 5^x$$

Original:  $x > 2/3$ . Inverse: all real numbers. Derivative: all real numbers.

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

#### 5. $f(x)$

$$= \frac{10^x + 4}{-4}$$

Swap x and y:

$$x = \frac{10^y + 4}{-4}$$

Isolate the exponential:

$$-4 \cdot x - 4 = 10^y$$

Take logarithm base 10:

$$y = \log_{10}(-4 \cdot x - 4)$$

$$g(x) = \log_{10}(-4 \cdot x - 4)$$

Differentiate the logarithmic inverse:

$$g' = \frac{-4}{(-4 \cdot x - 4) \cdot \ln 10}$$

Original: all real numbers. Inverse:  $x < -1$ . Derivative:  $x < -1$ .

#### 6. $f(x)$

$$= \frac{5 \cdot x - 9}{3}$$

Swap x and y:

$$x = \frac{5 \cdot y - 9}{3}$$

Multiply by the denominator:

$$3 \cdot x = 5 \cdot y - 9$$

Isolate y:

$$y = \frac{3 \cdot x + 9}{5}$$

$$g(x) = \frac{3 \cdot x + 9}{5}$$

Differentiate g directly:

$$g' = \frac{3}{5}$$

Original: all real numbers. Inverse: all real numbers. Derivative: all real numbers.

## Derivatives of Inverse Functions

## Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

7.  $f(x)$ 

$$= -2 \cdot (x+3)^3 - 1$$

Swap x and y:

$$x = -2 \cdot (y+3)^3 - 1$$

Subtract k and divide by a:

$$(y+3)^3 = \frac{x+1}{-2}$$

Take the selected real root:

$$y+3 = \sqrt[3]{\frac{x+1}{-2}}$$

Solve for y:

$$y = \sqrt[3]{\frac{x+1}{-2}} - 3$$

$$g(x) = \sqrt[3]{\frac{x+1}{-2}} - 3$$

Differentiate the explicit inverse:

$$g' = \frac{-1}{6} \cdot \left( \frac{x+1}{-2} \right)^{-\frac{2}{3}}$$

Original: all real numbers. Inverse: all real numbers. Derivative:  $x \neq -1$ .

The cube-root inverse exists at  $x = -1$ , but has no finite ordinary two-sided derivative there.

8.  $f(x)$ 

$$= \frac{-1 \cdot x + 1}{3 \cdot x + 3}$$

Swap x and y:

$$x = \frac{-1 \cdot y + 1}{3 \cdot y + 3}$$

Cross-multiply:

$$x \cdot (3 \cdot y + 3) = -1 \cdot y + 1$$

Collect the y terms:

$$(3 \cdot x + 1) \cdot y = 1 - 3 \cdot x$$

Solve for y:

$$y = \frac{1 - 3 \cdot x}{3 \cdot x + 1}$$

$$g(x) = \frac{1 - 3 \cdot x}{3 \cdot x + 1}$$

Apply the quotient rule:

$$g' = \frac{-3 \cdot (3 \cdot x + 1) - 1 \cdot 3 \cdot (1 - 3 \cdot x)}{(3 \cdot x + 1)^2}$$

Collapse the numerator to the determinant:

$$g' = \frac{-6}{(3 \cdot x + 1)^2}$$

Original:  $x \neq -1$ . Inverse:  $x \neq -1/3$ . Derivative:  $x \neq -1/3$ .

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f^{-1}$ , then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

#### 9. $f(x)$

$$= \log_5(4 \cdot x - 4)$$

Swap x and y:

$$x = \log_5(4 \cdot y - 4)$$

Exponentiate:

$$5^x = 4 \cdot y - 4$$

Isolate y:

$$y = \frac{5^x + 4}{4}$$

$$g(x) = \frac{5^x + 4}{4}$$

Differentiate the exponential inverse:

$$g' = \frac{1}{4} \cdot \ln 5 \cdot 5^x$$

Original:  $x > 1$ . Inverse: all real numbers. Derivative: all real numbers.

#### 10. $f(x)$

$$= \frac{10^x - 3}{5}$$

Swap x and y:

$$x = \frac{10^y - 3}{5}$$

Isolate the exponential:

$$5 \cdot x + 3 = 10^y$$

Take logarithm base 10:

$$y = \log_{10}(5 \cdot x + 3)$$

$$g(x) = \log_{10}(5 \cdot x + 3)$$

Differentiate the logarithmic inverse:

$$g' = \frac{5}{(5 \cdot x + 3) \cdot \ln 10}$$

Original: all real numbers. Inverse:  $x > -3/5$ . Derivative:  $x > -3/5$ .

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f^{-1}$ , then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

11.  $f(x)$

$$= \frac{4 \cdot x - 1}{3}$$

Swap  $x$  and  $y$ :

$$x = \frac{4 \cdot y - 1}{3}$$

Multiply by the denominator:

$$3 \cdot x = 4 \cdot y - 1$$

Isolate  $y$ :

$$y = \frac{3 \cdot x + 1}{4}$$

$$g(x) = \frac{3 \cdot x + 1}{4}$$

Differentiate  $g$  directly:

$$g' = \frac{3}{4}$$

Original: all real numbers. Inverse: all real numbers. Derivative: all real numbers.

12.  $f(x)$

$$= 3 \cdot (x + 4)^4 + 1$$

Swap  $x$  and  $y$ :

$$x = 3 \cdot (y + 4)^4 + 1$$

Subtract  $k$  and divide by  $a$ :

$$(y + 4)^4 = \frac{x - 1}{3}$$

Take the selected real root:

$$y + 4 = -1 \cdot \sqrt[4]{\frac{x - 1}{3}}$$

Solve for  $y$ :

$$y = -1 \cdot \sqrt[4]{\frac{x - 1}{3}} - 4$$

$$g(x) = -1 \cdot \sqrt[4]{\frac{x - 1}{3}} - 4$$

Differentiate the explicit inverse:

$$g' = \frac{-1}{12} \left( \frac{x - 1}{3} \right)^{-\frac{3}{4}}$$

Original:  $x \leq -4$ . Inverse:  $x \geq 1$ . Derivative:  $x > 1$ .

The inverse exists at  $x = 1$ , but its derivative is not finite there.

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

#### 13. $f(x)$

$$= \frac{-2}{x - 3}$$

Swap x and y:

$$x = \frac{-2}{y - 3}$$

Cross-multiply:

$$x \cdot (y - 3) = -2$$

Collect the y terms:

$$x \cdot y = -2 + 3 \cdot x$$

Solve for y:

$$y = \frac{-2 + 3 \cdot x}{x}$$

$$g(x) = \frac{-2 + 3 \cdot x}{x}$$

Apply the quotient rule:

$$g' = \frac{3 \cdot x - 1 \cdot (-2 + 3 \cdot x)}{x^2}$$

Collapse the numerator to the determinant:

$$g' = \frac{2}{x^2}$$

Original:  $x \neq 3$ . Inverse:  $x \neq 0$ . Derivative:  $x \neq 0$ .

#### 14. $f(x)$

$$= \log_3(5 \cdot x + 5)$$

Swap x and y:

$$x = \log_3(5 \cdot y + 5)$$

Exponentiate:

$$3^x = 5 \cdot y + 5$$

Isolate y:

$$y = \frac{3^x - 5}{5}$$

$$g(x) = \frac{3^x - 5}{5}$$

Differentiate the exponential inverse:

$$g' = \frac{1}{5} \cdot \ln 3 \cdot 3^x$$

Original:  $x > -1$ . Inverse: all real numbers. Derivative: all real numbers.

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f^{-1}$ , then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

#### 15. $f(x)$

$$= \frac{5^x - 2}{-2}$$

Swap x and y:

$$x = \frac{5^y - 2}{-2}$$

Isolate the exponential:

$$-2 \cdot x + 2 = 5^y$$

Take logarithm base 5:

$$y = \log_5(-2 \cdot x + 2)$$

$$g(x) = \log_5(-2 \cdot x + 2)$$

Differentiate the logarithmic inverse:

$$g' = \frac{-2}{(-2 \cdot x + 2) \cdot \ln 5}$$

Original: all real numbers. Inverse:  $x < 1$ . Derivative:  $x < 1$ .

#### 16. $f(x)$

$$= \frac{6 \cdot x + 8}{5}$$

Swap x and y:

$$x = \frac{6 \cdot y + 8}{5}$$

Multiply by the denominator:

$$5 \cdot x = 6 \cdot y + 8$$

Isolate y:

$$y = \frac{5 \cdot x - 8}{6}$$

$$g(x) = \frac{5 \cdot x - 8}{6}$$

Differentiate g directly:

$$g' = \frac{5}{6}$$

Original: all real numbers. Inverse: all real numbers. Derivative: all real numbers.

## Derivatives of Inverse Functions

## Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

17.  $f(x)$ 

$$= -1 \cdot (x+2)^2 - 1$$

Swap x and y:

$$x = -1 \cdot (y+2)^2 - 1$$

Subtract k and divide by a:

$$(y+2)^2 = \frac{x+1}{-1}$$

Take the selected real root:

$$y+2 = -1 \cdot \sqrt{\frac{x+1}{-1}}$$

Solve for y:

$$y = -1 \cdot \sqrt{\frac{x+1}{-1}} - 2$$

$$g(x) = -1 \cdot \sqrt{\frac{x+1}{-1}} - 2$$

Differentiate the explicit inverse:

$$g' = \frac{1}{2} \cdot \left( \frac{x+1}{-1} \right)^{-\frac{1}{2}}$$

Original:  $x \leq -2$ . Inverse:  $x \leq -1$ . Derivative:  $x < -1$ .

The inverse exists at  $x = -1$ , but its derivative is not finite there.

18.  $f(x)$ 

$$= \frac{4 \cdot x - 3}{2 \cdot x}$$

Swap x and y:

$$x = \frac{4 \cdot y - 3}{2 \cdot y}$$

Cross-multiply:

$$x \cdot 2 \cdot y = 4 \cdot y - 3$$

Collect the y terms:

$$(2 \cdot x - 4) \cdot y = -3$$

Solve for y:

$$y = \frac{-3}{2 \cdot x - 4}$$

$$g(x) = \frac{-3}{2 \cdot x - 4}$$

Apply the quotient rule:

$$g' = \frac{-1 \cdot 2 \cdot -3}{(2 \cdot x - 4)^2}$$

Collapse the numerator to the determinant:

$$g' = \frac{6}{(2 \cdot x - 4)^2}$$

Original:  $x \neq 0$ . Inverse:  $x \neq 2$ . Derivative:  $x \neq 2$ .

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f^{-1}$ , then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

#### 19. $f(x)$

$$= \log_{10}(-2 \cdot x - 2)$$

Swap x and y:

$$x = \log_{10}(-2 \cdot y - 2)$$

Exponentiate:

$$10^x = -2 \cdot y - 2$$

Isolate y:

$$y = \frac{10^x + 2}{-2}$$

$$g(x) = \frac{10^x + 2}{-2}$$

Differentiate the exponential inverse:

$$g' = \frac{-1}{2} \cdot \ln 10 \cdot 10^x$$

Original:  $x < -1$ . Inverse: all real numbers. Derivative: all real numbers.

#### 20. $f(x)$

$$= \frac{2^x - 4}{5}$$

Swap x and y:

$$x = \frac{2^y - 4}{5}$$

Isolate the exponential:

$$5 \cdot x + 4 = 2^y$$

Take logarithm base 2:

$$y = \log_2(5 \cdot x + 4)$$

$$g(x) = \log_2(5 \cdot x + 4)$$

Differentiate the logarithmic inverse:

$$g' = \frac{5}{(5 \cdot x + 4) \cdot \ln 2}$$

Original: all real numbers. Inverse:  $x > -4/5$ . Derivative:  $x > -4/5$ .

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

21.  $f(x)$

$$= \frac{6 \cdot x - 8}{3}$$

Swap x and y:

$$x = \frac{6 \cdot y - 8}{3}$$

Multiply by the denominator:

$$3 \cdot x = 6 \cdot y - 8$$

Isolate y:

$$y = \frac{3 \cdot x + 8}{6}$$

$$g(x) = \frac{3 \cdot x + 8}{6}$$

Differentiate g directly:

$$g' = \frac{1}{2}$$

Original: all real numbers. Inverse: all real numbers. Derivative: all real numbers.

22.  $f(x)$

$$= (x - 1)^3$$

Swap x and y:

$$x = (y - 1)^3$$

Subtract k and divide by a:

$$(y - 1)^3 = x$$

Take the selected real root:

$$y - 1 = \sqrt[3]{x}$$

Solve for y:

$$y = \sqrt[3]{x} + 1$$

$$g(x) = \sqrt[3]{x} + 1$$

Differentiate the explicit inverse:

$$g' = \frac{1}{3} \cdot x^{-\frac{2}{3}}$$

Original: all real numbers. Inverse: all real numbers. Derivative:  $x \neq 0$ .

The cube-root inverse exists at  $x = 0$ , but has no finite ordinary two-sided derivative there.

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

23.  $f(x)$

$$= \frac{-2 \cdot x + 4}{x + 4}$$

Swap  $x$  and  $y$ :

$$x = \frac{-2 \cdot y + 4}{y + 4}$$

Cross-multiply:

$$x \cdot (y + 4) = -2 \cdot y + 4$$

Collect the  $y$  terms:

$$(x + 2) \cdot y = 4 - 4 \cdot x$$

Solve for  $y$ :

$$y = \frac{4 - 4 \cdot x}{x + 2}$$

$$g(x) = \frac{4 - 4 \cdot x}{x + 2}$$

Apply the quotient rule:

$$g' = \frac{-4 \cdot (x + 2) - 1 \cdot (4 - 4 \cdot x)}{(x + 2)^2}$$

Collapse the numerator to the determinant:

$$g' = \frac{-12}{(x + 2)^2}$$

Original:  $x \neq -4$ . Inverse:  $x \neq -2$ . Derivative:  $x \neq -2$ .

24.  $f(x)$

$$= \log_{10}(3 \cdot x - 5)$$

Swap  $x$  and  $y$ :

$$x = \log_{10}(3 \cdot y - 5)$$

Exponentiate:

$$10^x = 3 \cdot y - 5$$

Isolate  $y$ :

$$y = \frac{10^x + 5}{3}$$

$$g(x) = \frac{10^x + 5}{3}$$

Differentiate the exponential inverse:

$$g' = \frac{1}{3} \cdot \ln 10 \cdot 10^x$$

Original:  $x > 5/3$ . Inverse: all real numbers. Derivative: all real numbers.

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

25.  $f(x)$

$$= \frac{10^x + 4}{-2}$$

Swap x and y:

$$x = \frac{10^y + 4}{-2}$$

Isolate the exponential:

$$-2 \cdot x - 4 = 10^y$$

Take logarithm base 10:

$$y = \log_{10}(-2 \cdot x - 4)$$

$$g(x) = \log_{10}(-2 \cdot x - 4)$$

Differentiate the logarithmic inverse:

$$g' = \frac{-2}{(-2 \cdot x - 4) \cdot \ln 10}$$

Original: all real numbers. Inverse:  $x < -2$ . Derivative:  $x < -2$ .

26.  $f(x)$

$$= \frac{3 \cdot x + 4}{3}$$

Swap x and y:

$$x = \frac{3 \cdot y + 4}{3}$$

Multiply by the denominator:

$$3 \cdot x = 3 \cdot y + 4$$

Isolate y:

$$y = \frac{3 \cdot x - 4}{3}$$

$$g(x) = \frac{3 \cdot x - 4}{3}$$

Differentiate g directly:

$$g' = 1$$

Original: all real numbers. Inverse: all real numbers. Derivative: all real numbers.

## Derivatives of Inverse Functions

## Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

27.  $f(x)$ 

$$= 3 \cdot (x - 3)^4$$

Swap x and y:

$$x = 3 \cdot (y - 3)^4$$

Subtract k and divide by a:

$$(y - 3)^4 = \frac{x}{3}$$

Take the selected real root:

$$y - 3 = \sqrt[4]{\frac{x}{3}}$$

Solve for y:

$$y = \sqrt[4]{\frac{x}{3}} + 3$$

$$g(x) = \sqrt[4]{\frac{x}{3}} + 3$$

Differentiate the explicit inverse:

$$g' = \frac{1}{12} \cdot \left(\frac{x}{3}\right)^{-\frac{3}{4}}$$

Original:  $x \geq 3$ . Inverse:  $x \geq 0$ . Derivative:  $x > 0$ .The inverse exists at  $x = 0$ , but its derivative is not finite there.28.  $f(x)$ 

$$= \frac{x+5}{3 \cdot x+1}$$

Swap x and y:

$$x = \frac{y+5}{3 \cdot y+1}$$

Cross-multiply:

$$x \cdot (3 \cdot y + 1) = y + 5$$

Collect the y terms:

$$(3 \cdot x - 1) \cdot y = 5 - 1 \cdot x$$

Solve for y:

$$y = \frac{5 - 1 \cdot x}{3 \cdot x - 1}$$

$$g(x) = \frac{5 - 1 \cdot x}{3 \cdot x - 1}$$

Apply the quotient rule:

$$g' = \frac{-1 \cdot (3 \cdot x - 1) - 1 \cdot 3 \cdot (5 - 1 \cdot x)}{(3 \cdot x - 1)^2}$$

Collapse the numerator to the determinant:

$$g' = \frac{-14}{(3 \cdot x - 1)^2}$$

Original:  $x \neq -1/3$ . Inverse:  $x \neq 1/3$ . Derivative:  $x \neq 1/3$ .

## Derivatives of Inverse Functions

### Worked Answer Key

Find the inverse  $g = f$  inverse, then differentiate the explicit inverse directly. State the original, inverse, and derivative domains.

29.  $f(x)$

$$= \log_5(5 \cdot x - 4)$$

Swap x and y:

$$x = \log_5(5 \cdot y - 4)$$

Exponentiate:

$$5^x = 5 \cdot y - 4$$

Isolate y:

$$y = \frac{5^x + 4}{5}$$

$$g(x) = \frac{5^x + 4}{5}$$

Differentiate the exponential inverse:

$$g' = \frac{1}{5} \cdot \ln 5 \cdot 5^x$$

Original:  $x > 4/5$ . Inverse: all real numbers. Derivative: all real numbers.

30.  $f(x)$

$$= \frac{5^x - 3}{-2}$$

Swap x and y:

$$x = \frac{5^y - 3}{-2}$$

Isolate the exponential:

$$-2 \cdot x + 3 = 5^y$$

Take logarithm base 5:

$$y = \log_5(-2 \cdot x + 3)$$

$$g(x) = \log_5(-2 \cdot x + 3)$$

Differentiate the logarithmic inverse:

$$g' = \frac{-2}{(-2 \cdot x + 3) \cdot \ln 5}$$

Original: all real numbers. Inverse:  $x < 3/2$ . Derivative:  $x < 3/2$ .