

# Math Sheet

Name \_\_\_\_\_ Score \_\_\_\_\_ Teacher \_\_\_\_\_ Date \_\_\_\_\_

## Integration by Parts

Choose u and dv, apply the parts identity, continue any polynomial reduction, and include + C.

1.

$$\int \frac{3}{2} \cdot (-1+x) \cdot \sin(-3+x) dx$$

$$u = -1+x$$

$$dv = \frac{3}{2} \cdot \sin(-3+x) \cdot dx$$

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4.

$$\int -1 \cdot \arctan(1-3 \cdot x) dx$$

$$u = \arctan(1-3 \cdot x)$$

$$dv = -1 \cdot dx$$

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2.

$$\int 2 \cdot (3-3 \cdot x-2 \cdot x^2) \cdot \cos(-3+2 \cdot x) dx$$

$$u = 3-3 \cdot x-2 \cdot x^2$$

$$dv = 2 \cdot \cos(-3+2 \cdot x) \cdot dx$$

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5.

$$\int \frac{3}{2} \cdot \arcsin(-2+2 \cdot x) dx$$

$$u = \arcsin(-2+2 \cdot x)$$

$$dv = \frac{3}{2} \cdot dx$$

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3.

$$\int (2-x)^0 \cdot \ln(2-x) dx$$

$$u = \ln(2-x)$$

$$dv = (2-x)^0 \cdot dx$$

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6.

$$\int -1 \cdot (x^2-2 \cdot x^3) \cdot e^{-3+x} dx$$

$$u = x^2-2 \cdot x^3$$

$$dv = -1 \cdot e^{-3+x} \cdot dx$$

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# Math Sheet

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## Integration by Parts

### Worked Answer Key

Choose u and dv, apply the parts identity, continue any polynomial reduction, and include + C.

1. Original:  $\int \frac{3}{2}(-1+x) \sin(-3+x) dx$

Step 1  $u = -1+x$

$$dv = \frac{3}{2} \sin(-3+x) dx$$

$$du = dx$$

$$v = \frac{-3}{2} \cos(-3+x)$$

$$\text{Identity (+ outer), I1} = (-1+x) \cdot \frac{-3}{2} \cos(-3+x) - \text{R1}$$

$$\text{R1} = \int \frac{-3}{2} \cos(-3+x) dx$$

Continue with the next integration-by-parts stage.

Step 2  $u = 1$

$$dv = \frac{-3}{2} \cos(-3+x) dx$$

$$du = 0 dx$$

$$v = \frac{-3}{2} \sin(-3+x)$$

$$\text{Identity (- outer), I2} = - \frac{3}{2} \sin(-3+x) - \text{R2}$$

$$\text{R2} = \int 0 dx$$

Residual primitive: 0

$$\text{Collected: } \frac{3}{2} \sin(-3+x) + \left( \frac{3}{2} - \frac{3}{2}x \right) \cos(-3+x)$$

$$\text{Answer: } \frac{3}{2} \sin(-3+x) + \left( \frac{3}{2} - \frac{3}{2}x \right) \cos(-3+x) + C$$

Original integrand and parts identity are valid for all real x.

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## Integration by Parts

### Worked Answer Key

Choose u and dv, apply the parts identity, continue any polynomial reduction, and include + C.

2. Original:  $\int 2(3 - 3x - 2x^2) \cos(-3 + 2x) dx$

Step 1  $u = 3 - 3x - 2x^2$

$dv = 2 \cos(-3 + 2x) \cdot dx$

$du = (-3 - 4x) \cdot dx$

$v = \sin(-3 + 2x)$

Identity (+ outer), I1 =  $(3 - 3x - 2x^2) \sin(-3 + 2x) - R1$

$R1 = \int \sin(-3 + 2x) (-3 - 4x) dx$

Continue with the next integration-by-parts stage.

Step 2  $u = -3 - 4x$

$dv = \sin(-3 + 2x) \cdot dx$

$du = -4 \cdot dx$

$v = \frac{-1}{2} \cos(-3 + 2x)$

Identity (- outer), I2 =  $(-3 - 4x) \cdot \frac{-1}{2} \cos(-3 + 2x) - R2$

$R2 = \int \frac{-1}{2} \cos(-3 + 2x) \cdot -4 dx$

Continue with the next integration-by-parts stage.

Step 3  $u = -4$

$dv = \frac{-1}{2} \cos(-3 + 2x) \cdot dx$

$du = 0 \cdot dx$

$v = \frac{-1}{4} \sin(-3 + 2x)$

Identity (+ outer), I3 =  $-4 \cdot \frac{-1}{4} \sin(-3 + 2x) - R3$

$R3 = \int 0 dx$

Residual primitive: 0

Collected:  $(4 - 3x - 2x^2) \sin(-3 + 2x) + \left(-\frac{3}{2} - 2x\right) \cos(-3 + 2x)$

Answer:  $(4 - 3x - 2x^2) \sin(-3 + 2x) + \left(-\frac{3}{2} - 2x\right) \cos(-3 + 2x) + C$

Original integrand and parts identity are valid for all real x.

3. Original:  $\int (2 - x)^0 \cdot \ln(2 - x) dx$

Step 1  $u = \ln(2 - x)$

$dv = (2 - x)^0 \cdot dx$

$du = \frac{-1}{2 - x} \cdot dx$

$v = -1 \cdot (2 - x)$

Identity (+ outer), I1 =  $\ln(2 - x) \cdot -1 \cdot (2 - x) - R1$

$R1 = \int (2 - x)^0 dx$

Residual primitive:  $-1 \cdot (2 - x)$

Answer:  $-(2 - x) \cdot (\ln(2 - x) - 1) + C$

Original and parts domain:  $2 - x > 0$

Original integrand and parts identity require  $h(x) > 0$ . The answer holds on that open interval.

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## Integration by Parts

### Worked Answer Key

Choose u and dv, apply the parts identity, continue any polynomial reduction, and include + C.

4. Original:  $\int -1 \cdot \arctan(1 - 3 \cdot x) dx$

Step 1 u =  $\arctan(1 - 3 \cdot x)$

dv =  $-1 \cdot dx$

du =  $\frac{-3}{1 + (1 - 3 \cdot x)^2} \cdot dx$

v =  $\frac{1}{3} (1 - 3 \cdot x)$

Identity (+ outer), I1 =  $\arctan(1 - 3 \cdot x) \cdot \frac{1}{3} (1 - 3 \cdot x) - R1$

R1 =  $\int \frac{-1 \cdot (1 - 3 \cdot x)}{1 + (1 - 3 \cdot x)^2} dx$

Residual primitive:  $\frac{1}{6} \cdot \ln(1 + (1 - 3 \cdot x)^2)$

Convenient constant choice: since  $h'(x) = a$ ,  $d[(k/a)h(x)]/dx = k$ .

Answer:  $\frac{1}{3} \left( (1 - 3 \cdot x) \cdot \arctan(1 - 3 \cdot x) - \frac{1}{2} \cdot \ln(1 + (1 - 3 \cdot x)^2) \right) + C$

Original integrand and parts identity are valid for all real x.

5. Original:  $\int \frac{3}{2} \cdot \arcsin(-2 + 2 \cdot x) dx$

Step 1 u =  $\arcsin(-2 + 2 \cdot x)$

dv =  $\frac{3}{2} \cdot dx$

du =  $\frac{2}{\sqrt{1 - (-2 + 2 \cdot x)^2}} \cdot dx$

v =  $\frac{3}{4} (-2 + 2 \cdot x)$

Identity (+ outer), I1 =  $\arcsin(-2 + 2 \cdot x) \cdot \frac{3}{4} (-2 + 2 \cdot x) - R1$

R1 =  $\int \frac{\frac{3}{2} (-2 + 2 \cdot x)}{\sqrt{1 - (-2 + 2 \cdot x)^2}} dx$

Residual primitive:  $\frac{-3}{4} \sqrt{1 - (-2 + 2 \cdot x)^2}$

Convenient constant choice: since  $h'(x) = a$ ,  $d[(k/a)h(x)]/dx = k$ .

Answer:  $\frac{3}{4} \left( (-2 + 2 \cdot x) \cdot \arcsin(-2 + 2 \cdot x) + \sqrt{1 - (-2 + 2 \cdot x)^2} \right) + C$

Original real domain:  $|-2 + 2 \cdot x| \leq 1$

Parts identity domain:  $|-2 + 2 \cdot x| < 1$

Original real integrand:  $|h(x)| \leq 1$ . Parts identity:  $|h(x)| < 1$ ; finite endpoint primitive values are allowed.

## Integration by Parts

### Worked Answer Key

Choose u and dv, apply the parts identity, continue any polynomial reduction, and include + C.

6. Original:  $\int -1 \cdot (x^2 - 2 \cdot x^3) \cdot e^{-3+x} dx$

Step 1  $u = x^2 - 2 \cdot x^3$

$dv = -1 \cdot e^{-3+x} \cdot dx$

$du = (2 \cdot x - 6 \cdot x^2) \cdot dx$

$v = -1 \cdot e^{-3+x}$

Identity (+ outer), I1 =  $(x^2 - 2 \cdot x^3) \cdot -1 \cdot e^{-3+x} - R1$

$R1 = \int -1 \cdot e^{-3+x} (2 \cdot x - 6 \cdot x^2) dx$

Continue with the next integration-by-parts stage.

Step 2  $u = 2 \cdot x - 6 \cdot x^2$

$dv = -1 \cdot e^{-3+x} \cdot dx$

$du = (2 - 12 \cdot x) \cdot dx$

$v = -1 \cdot e^{-3+x}$

Identity (- outer), I2 =  $(2 \cdot x - 6 \cdot x^2) \cdot -1 \cdot e^{-3+x} - R2$

$R2 = \int -1 \cdot e^{-3+x} (2 - 12 \cdot x) dx$

Continue with the next integration-by-parts stage.

Step 3  $u = 2 - 12 \cdot x$

$dv = -1 \cdot e^{-3+x} \cdot dx$

$du = -12 \cdot dx$

$v = -1 \cdot e^{-3+x}$

Identity (+ outer), I3 =  $(2 - 12 \cdot x) \cdot -1 \cdot e^{-3+x} - R3$

$R3 = \int -1 \cdot e^{-3+x} \cdot -12 dx$

Continue with the next integration-by-parts stage.

Step 4  $u = -12$

$dv = -1 \cdot e^{-3+x} \cdot dx$

$du = 0 \cdot dx$

$v = -1 \cdot e^{-3+x}$

Identity (- outer), I4 =  $-12 \cdot -1 \cdot e^{-3+x} - R4$

$R4 = \int 0 dx$

Residual primitive: 0

Collected:  $(-14 + 14 \cdot x - 7 \cdot x^2 + 2 \cdot x^3) \cdot e^{-3+x}$

Answer:  $(-14 + 14 \cdot x - 7 \cdot x^2 + 2 \cdot x^3) \cdot e^{-3+x} + C$

Original integrand and parts identity are valid for all real x.