

Separable Initial-Value Problems

Separate variables, retain every equilibrium solution, apply the initial condition, and state the maximal interval containing the initial point.

1. Solve $y' = 2y$, $y(2) = -2$. Include equilibrium solutions and the maximal interval.

2. Solve $y' = 1(y-0)^2$, $y(0) = 1$. Include equilibrium solutions and the maximal interval.

Separable Initial-Value Problems

Separate variables, retain every equilibrium solution, apply the initial condition, and state the maximal interval containing the initial point.

3. Solve $y' = -3(y-2)^2$, $y(2) = 2$. Include equilibrium solutions and the maximal interval.

4. Solve $y' = -2y$, $y(0) = 0$. Include equilibrium solutions and the maximal interval.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Separable Initial-Value Problems

Worked Answer Key

Separate variables, retain every equilibrium solution, apply the initial condition, and state the maximal interval containing the initial point.

1. Solve $y' = 2y$, $y(2) = -2$. Include equilibrium solutions and the maximal interval.

Answer: $y = -2e^{2(x-2)}$ on $(-\infty, \infty)$.

1. Separate nonzero branches: $dy/y = 2 dx$; integrate to $\ln|y| = 2x + C$.
2. The divided-out equilibrium $y = 0$ must also be retained.
3. Apply $y(2) = -2$; the constant gives $y = -2e^{2(x-2)}$.
4. The equation and solution are defined for every real x , so the maximal interval is $(-\infty, \infty)$.

2. Solve $y' = 1(y-0)^2$, $y(0) = 1$. Include equilibrium solutions and the maximal interval.

Answer: $y = 0 + 1/(1/1 - 1(x-0))$ on $(-\infty, 1)$. The DE equilibrium $y = 0$ does not meet this IVP.

1. The differential equation has equilibrium $y = 0$; it does not satisfy $y(0) = 1$. For nonconstant branches, $dy/(y-0)^2 = 1 dx$.
2. Integrate: $-1/(y-0) = 1x + C$.
3. Using $y(0)-0 = 1$ gives $1/(y-0) = 1/1 - 1(x-0)$.
4. The denominator vanishes exactly at $x = 1$; the maximal interval containing $x_0 = 0$ is $(-\infty, 1)$.

Separable Initial-Value Problems

Worked Answer Key

Separate variables, retain every equilibrium solution, apply the initial condition, and state the maximal interval containing the initial point.

3. Solve $y' = -3(y-2)^2$, $y(2) = 2$. Include equilibrium solutions and the maximal interval.

Answer: $y = 2$ on $(-\infty, \infty)$.

1. The constant branch $y = 2$ makes both sides zero.
2. Separating after division would lose this equilibrium, so retain it explicitly.
3. The initial value $y(2) = 2$ selects that equilibrium branch.
4. It is defined for all real x ; the maximal interval is $(-\infty, \infty)$.

4. Solve $y' = -2y$, $y(0) = 0$. Include equilibrium solutions and the maximal interval.

Answer: $y = 0$ on $(-\infty, \infty)$.

1. Separate nonzero branches: $dy/y = -2 dx$; integrate to $\ln|y| = -2x + C$.
2. The divided-out equilibrium $y = 0$ must also be retained.
3. Apply $y(0) = 0$; uniqueness selects the equilibrium branch.
4. The equation and solution are defined for every real x , so the maximal interval is $(-\infty, \infty)$.