

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Improper Integrals

Rewrite each integral as the required one-sided limit, split at every interior singularity, and classify convergence before evaluating. Do not use principal values.

1. Classify and, if convergent, evaluate integral from 0 to 3 of $3/\sqrt{x}$ dx.

2. Classify and, if convergent, evaluate integral from -4 to 4 of $3/|x|$ dx. Split at $x = 0$; do not use a principal value.

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3. Classify and, if convergent, evaluate integral from 0 to 3 of $3/x$ dx.

4. Classify and, if convergent, evaluate integral from 4 to infinity of $2/x^2$ dx.

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Improper Integrals

Rewrite each integral as the required one-sided limit, split at every interior singularity, and classify convergence before evaluating. Do not use principal values.

5. Classify and, if convergent, evaluate integral from -3 to 3 of $\frac{3}{\sqrt{|x|}}$ dx. Split at $x = 0$; do not use a principal value.

6. Classify and, if convergent, evaluate integral from 3 to infinity of $\frac{3}{x^1}$ dx.

Improper Integrals

Worked Answer Key

Rewrite each integral as the required one-sided limit, split at every interior singularity, and classify convergence before evaluating. Do not use principal values.

1. Classify and, if convergent, evaluate integral from 0 to 3 of $3/\sqrt{x}$ dx.

Answer: Convergent; exact value $6\sqrt{3}$.

1. Use the one-sided limit $\lim_{t \rightarrow 0^+} [6\sqrt{x}]$ from t to 3.
2. The t-term tends to 0 because $p = 1/2 < 1$.
3. Combine the finite side values to get $6\sqrt{3}$.

2. Classify and, if convergent, evaluate integral from -4 to 4 of $3/|x|$ dx. Split at $x = 0$; do not use a principal value.

Answer: Divergent to infinity.

1. Split at 0 into left and right limits; by symmetry each is $\lim_{t \rightarrow 0^+} [3\ln(x)]$ from t to 4.
2. $\ln(t)$ tends to negative infinity, so each required one-sided integral diverges.
3. Do not cancel the two divergent sides or assign a principal value.

Improper Integrals

Worked Answer Key

Rewrite each integral as the required one-sided limit, split at every interior singularity, and classify convergence before evaluating. Do not use principal values.

3. Classify and, if convergent, evaluate integral from 0 to 3 of $3/x$ dx.

Answer: Divergent to infinity.

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2. $\ln(t)$ tends to negative infinity, so each required one-sided integral diverges.
3. Do not cancel the two divergent sides or assign a principal value.

4. Classify and, if convergent, evaluate integral from 4 to infinity of $2/x^2$ dx.

Answer: Convergent; exact value $1/2$.

1. Write $\lim_{b \rightarrow \infty} [2/-1x^{-1}]$ from 4 to b .
2. Since $p = 2 > 1$, the b -term tends to 0.
3. The remaining lower-end value is $1/2$.

Improper Integrals

Worked Answer Key

Rewrite each integral as the required one-sided limit, split at every interior singularity, and classify convergence before evaluating. Do not use principal values.

5. Classify and, if convergent, evaluate integral from -3 to 3 of $\frac{3}{\sqrt{|x|}}$ dx. Split at $x = 0$; do not use a principal value.

Answer: Convergent; exact value $12\sqrt{3}$.

1. Split at 0 into left and right limits; by symmetry each is $\lim_{t \rightarrow 0^+} [6\sqrt{t}]$ from t to 3.
2. The t -term tends to 0 because $p = 1/2 < 1$.
3. Combine the finite side values to get $12\sqrt{3}$.

6. Classify and, if convergent, evaluate integral from 3 to infinity of $\frac{3}{x^1}$ dx.

Answer: Divergent to infinity.

1. Write $\lim_{b \rightarrow \infty} [3 \ln(x)]$ from 3 to b .
2. Since $p = 1$, $\ln(b)$ grows without bound.
3. Therefore the ordinary improper integral has no finite value.