

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Horizontal Tangents

Differentiate each function, solve $f'(x) = 0$, enforce the original domain, and give every exact coordinate $(x, f(x))$. Trigonometric tasks use $0 \leq x < 2\pi$ radians.

1. Find all horizontal tangent points.

$$f(x) = x^3 + x$$

Domain: All real x.

2. Find all horizontal tangent points.

$$f(x) = \frac{x^2 + 2}{x}$$

Domain: x must not equal 0.

3. Find all horizontal tangent points for $0 \leq x < 2\pi$.

$$f(x) = -3 \cdot \sec(x) + 1$$

Domain: $\cos(x)$ must be nonzero.

4. Find all horizontal tangent points.

$$f(x) = x^3$$

Domain: All real x.

5. Find all horizontal tangent points.

$$f(x) = \frac{x^2 - 1}{x}$$

Domain: x must not equal 0.

6. Find all horizontal tangent points for $0 \leq x < 2\pi$.

$$f(x) = 4 \cdot \csc\left(-2 \cdot x + \frac{1}{2} \cdot \pi\right)$$

Domain: $\sin(-2x + 1/2\pi)$ must be nonzero.

Horizontal Tangents

Differentiate each function, solve $f'(x) = 0$, enforce the original domain, and give every exact coordinate $(x, f(x))$. Trigonometric tasks use $0 \leq x < 2\pi$ radians.

7. Find all horizontal tangent points.

$f(x) = x^3 - 2 \cdot x + 1$
Domain: All real x.

8. Find all horizontal tangent points.

$f(x) = \frac{x^2 - 2 \cdot x + 1}{x - 1}$

Domain: x must not equal 1.

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Horizontal Tangents

Worked Answer Key

Differentiate each function, solve $f'(x) = 0$, enforce the original domain, and give every exact coordinate $(x, f(x))$. Trigonometric tasks use $0 \leq x < 2\pi$ radians.

1. Differentiate and solve.

$$f(x) = x^3 + x$$

$$f'(x) = 3 \cdot x^2 + 1$$

The derivative has no real zeros.

No horizontal tangent points.

2. Differentiate and solve.

$$f(x) = \frac{x^2 + 2}{x}$$

$$f'(x) = \frac{x^2 - 2}{x^2}$$

Complete derivative roots: $-1\sqrt{2}, \sqrt{2}$.

$$(-1\sqrt{2}, -2\sqrt{2}) \quad (\sqrt{2}, 2\sqrt{2})$$

3. Differentiate and solve.

$$f(x) = -3 \cdot \sec(x) + 1$$

$$f'(x) = -3 \cdot \sec(x) \cdot \tan(x)$$

General stationary solutions: $x = (0+k*2)\pi, y = -2$; $x = (1+k*2)\pi, y = 4$, k integer.

Pole positions in the requested window are excluded by the original trig domain.

$$(0, -2) \quad (\pi, 4)$$

4. Differentiate and solve.

$$f(x) = x^3$$

$$f'(x) = 3 \cdot x^2$$

Complete derivative roots: 0 (multiplicity 2).

$$(0, 0)$$

5. Differentiate and solve.

$$f(x) = \frac{x^2 - 1}{x}$$

$$f'(x) = \frac{x^2 + 1}{x^2}$$

The derivative has no real zeros.

No horizontal tangent points.

6. Differentiate and solve.

$$f(x) = 4 \cdot \csc\left(-2 \cdot x + \frac{1}{2} \cdot \pi\right)$$

$$f'(x) = 8 \cdot \csc\left(-2 \cdot x + \frac{1}{2} \cdot \pi\right) \cdot \cot\left(-2 \cdot x + \frac{1}{2} \cdot \pi\right)$$

General stationary solutions: $x = (0+k*1)\pi, y = 4$; $x = (1/2+k*1)\pi, y = -4$, k integer.

Pole positions in the requested window are excluded by the original trig domain.

$$(0, 4) \quad \left(\frac{1}{2} \cdot \pi, -4\right) \quad (\pi, 4) \quad \left(\frac{3}{2} \cdot \pi, -4\right)$$

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Worked Answer Key

Differentiate each function, solve $f'(x) = 0$, enforce the original domain, and give every exact coordinate $(x, f(x))$. Trigonometric tasks use $0 \leq x < 2\pi$ radians.

7. Differentiate and solve.

$$f(x) = x^3 - 2x + 1$$

$$f'(x) = 3x^2 - 2$$

Complete derivative roots: $-1/3\sqrt{6}$, $1/3\sqrt{6}$.

$$\left(-\frac{1}{3}\sqrt{6}, \frac{4}{9}\sqrt{6} + 1\right) \quad \left(\frac{1}{3}\sqrt{6}, -\frac{4}{9}\sqrt{6} + 1\right)$$

8. Differentiate and solve.

$$f(x) = \frac{x^2 - 2x + 1}{x - 1}$$

$$f'(x) = \frac{x^2 - 2x + 1}{(x - 1)^2}$$

Complete derivative roots: 1 (multiplicity 2).

Exclude the derivative root at the original denominator hole.

No horizontal tangent points.