

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

General Differential Equations

Find the general solution on an interval of the original domain. Include one arbitrary constant C.
The arbitrary constant C may differ between connected intervals separated by an excluded point or trigonometric pole.

1.

$$\frac{dy}{dx} = -1$$

General solution: _____

2.

$$\frac{dy}{dx} = \frac{-8}{x}$$

General solution: _____

3.

$$\frac{dy}{dx} = \frac{1}{2} \cdot \cos(2 + 2 \cdot x)$$

General solution: _____

4.

$$\frac{dy}{dx} = -8 \cdot x^3 + 9 \cdot x^2$$

General solution: _____

5.

$$\frac{dy}{dx} = \frac{-5}{x^5}$$

General solution: _____

6.

$$\frac{dy}{dx} = \frac{1}{2} \cdot \sin(1 - 2 \cdot x)$$

General solution: _____

7.

$$\frac{dy}{dx} = \frac{-2}{1 \cdot x - 3}$$

General solution: _____

8.

$$\frac{dy}{dx} = -1 \cdot \sec(3 \cdot x)^2$$

General solution: _____

9.

$$\frac{dy}{dx} = -2 \cdot x^3 + 6 \cdot x^2 + 4$$

General solution: _____

10.

$$\frac{dy}{dx} = -1 \cdot \csc(3 + 3 \cdot x)^2$$

General solution: _____

General Differential Equations

Worked Answer Key

Find the general solution on an interval of the original domain. Include one arbitrary constant C.
The arbitrary constant C may differ between connected intervals separated by an excluded point or trigonometric pole.

1. General solution

$$\frac{dy}{dx} = -1$$

Integrate each power term with the power rule. Add 1 to each exponent. Divide each coefficient by its new exponent. Add one arbitrary constant C.

Solution: $y = -1 \cdot x + C$

Domain: all real x.

Check: $d/dx (-1 \cdot x) = -1$

Differentiate each primitive term: multiply by its power, then lower that power by 1.

2. General solution

$$\frac{dy}{dx} = \frac{-8}{x}$$

Use the reciprocal rule: the integral of $1/x$ is $\ln|x|$. Keep the coefficient outside the reciprocal rule. Use an absolute value because x may be positive or negative. Add one arbitrary constant C.

Solution: $y = -8 \cdot \ln(|x|) + C$

Domain: x is not 0; solve on a connected interval not crossing 0.

Check: $d/dx (-8 \cdot \ln(|x|)) = \frac{-8}{x}$

Differentiate $\ln|x|$: the chain factor is 1, so the recovered numerator is $-8/1$.

3. General solution

$$\frac{dy}{dx} = \frac{1}{2} \cdot \cos(2 + 2 \cdot x)$$

Use the matching trigonometric derivative pattern and divide by the inner slope. Match the cosine pattern. Apply the exact sign and inner-slope adjustment. Add one arbitrary constant C.

Solution: $y = \frac{1}{4} \cdot \sin(2 + 2 \cdot x) + C$

Domain: all real x.

Check: $d/dx \left(\frac{1}{4} \cdot \sin(2 + 2 \cdot x) \right) = \frac{1}{2} \cdot \cos(2 + 2 \cdot x)$

Multiply the primitive adjustment by the certified derivative factor $2/1$; the recovered coefficient is $1/2$.

4. General solution

$$\frac{dy}{dx} = -8 \cdot x^3 + 9 \cdot x^2$$

Integrate each power term with the power rule. Add 1 to each exponent. Divide each coefficient by its new exponent. Add one arbitrary constant C.

Solution: $y = -2 \cdot x^4 + 3 \cdot x^3 + C$

Domain: all real x.

Check: $d/dx (-2 \cdot x^4 + 3 \cdot x^3) = -8 \cdot x^3 + 9 \cdot x^2$

Differentiate each primitive term: multiply by its power, then lower that power by 1.

General Differential Equations

Worked Answer Key

Find the general solution on an interval of the original domain. Include one arbitrary constant C.
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5. General solution

$$\frac{dy}{dx} = \frac{-5}{x^5}$$

Rewrite the denominator as a negative power and apply the power rule. Write x^{-5} . Add 1 to the exponent and divide by it. Add one arbitrary constant C.

Solution: $y = \frac{5}{4} \cdot x^{-4} + C$

Domain: x is not 0; solve on a connected interval not crossing 0.

Check: $d/dx \left(\frac{5}{4} \cdot x^{-4} \right) = \frac{-5}{x^5}$

Multiply the primitive coefficient by its power and lower the power by 1 to recover x^{-5} .

6. General solution

$$\frac{dy}{dx} = \frac{1}{2} \cdot \sin(1 - 2 \cdot x)$$

Use the matching trigonometric derivative pattern and divide by the inner slope. Match the sine pattern. Apply the exact sign and inner-slope adjustment. Add one arbitrary constant C.

Solution: $y = \frac{1}{4} \cdot \cos(1 - 2 \cdot x) + C$

Domain: all real x.

Check: $d/dx \left(\frac{1}{4} \cdot \cos(1 - 2 \cdot x) \right) = \frac{1}{2} \cdot \sin(1 - 2 \cdot x)$

Multiply the primitive adjustment by the certified derivative factor 2/1; the recovered coefficient is 1/2.

7. General solution

$$\frac{dy}{dx} = \frac{-2}{-1 \cdot x - 3}$$

Use the affine reciprocal rule and divide by the inner slope. The inner derivative is -1. Multiply $\ln|ax+b|$ by c/a. Add one arbitrary constant C.

Solution: $y = \frac{2}{3} \cdot \ln(|-1 \cdot x - 3|) + C$

Domain: x is not -3/1; solve on one connected side of the pole.

Check: $d/dx \left(\frac{2}{3} \cdot \ln(|-1 \cdot x - 3|) \right) = \frac{-2}{-1 \cdot x - 3}$

Multiply the primitive coefficient by the inner slope -1; this recovers -2/3 over the affine denominator.

8. General solution

$$\frac{dy}{dx} = -1 \cdot \sec(3 \cdot x)^2$$

Use the matching trigonometric derivative pattern and divide by the inner slope. Match the secant-squared pattern. Apply the exact sign and inner-slope adjustment. Add one arbitrary constant C.

Solution: $y = -\frac{1}{3} \cdot \tan(3 \cdot x) + C$

Domain: $\cos(3x+0)$ is not 0; use one connected interval between poles.

Check: $d/dx \left(-\frac{1}{3} \cdot \tan(3 \cdot x) \right) = -1 \cdot \sec(3 \cdot x)^2$

Multiply the primitive adjustment by the certified derivative factor 3/1; the recovered coefficient is -1/1.

General Differential Equations

Worked Answer Key

Find the general solution on an interval of the original domain. Include one arbitrary constant C.
The arbitrary constant C may differ between connected intervals separated by an excluded point or trigonometric pole.

9. General solution

$$\frac{dy}{dx} = -2 \cdot x^3 + 6 \cdot x^2 + 4$$

Integrate each power term with the power rule. Add 1 to each exponent. Divide each coefficient by its new exponent. Add one arbitrary constant C.

Solution: $y = -\frac{1}{2} \cdot x^4 + 2 \cdot x^3 + 4 \cdot x + C$

Domain: all real x.

Check: $d/dx \left(-\frac{1}{2} \cdot x^4 + 2 \cdot x^3 + 4 \cdot x \right) = -2 \cdot x^3 + 6 \cdot x^2 + 4$

Differentiate each primitive term: multiply by its power, then lower that power by 1.

10. General solution

$$\frac{dy}{dx} = -1 \cdot \csc(3 + 3 \cdot x)^2$$

Use the matching trigonometric derivative pattern and divide by the inner slope. Match the cosecant-squared pattern. Apply the exact sign and inner-slope adjustment. Add one arbitrary constant C.

Solution: $y = \frac{1}{3} \cdot \cot(3 + 3 \cdot x) + C$

Domain: $\sin(3x+3)$ is not 0; use one connected interval between poles.

Check: $d/dx \left(\frac{1}{3} \cdot \cot(3 + 3 \cdot x) \right) = -1 \cdot \csc(3 + 3 \cdot x)^2$

Multiply the primitive adjustment by the certified derivative factor -3/1; the recovered coefficient is -1/1.