

## Exponential Growth and Decay

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

### 1. Growth model

Assume exponential growth. A cell culture measures 1200 cells at  $t=6$  hours and 2571 cells at  $t=13$  hours. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the modeled amount at  $t=16$  hours. Round the model estimate to the nearest whole cell. Write the model and show your work.

**Model:** \_\_\_\_\_

**Work / substitution:**

---

---

**Answer (cells):** \_\_\_\_\_

### 2. Decay model

Assume exponential decay. A laboratory sample measures 1240 grams at  $t=6$  hours and 670 grams at  $t=13$  hours. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the time  $t$  when the modeled amount reaches 469 grams. Round to 2 decimal places. Write the model and show your work.

**Model:** \_\_\_\_\_

**Work / substitution:**

---

---

**Answer (hours):** \_\_\_\_\_

### 3. Growth model

Assume exponential growth. A cell culture starts at 1200 cells and increases continuously at 4% per hour. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the modeled amount at  $t=16$  hours. Round the model estimate to the nearest whole cell. Write the model and show your work.

**Model:** \_\_\_\_\_

**Work / substitution:**

---

---

**Answer (cells):** \_\_\_\_\_

## Exponential Growth and Decay

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

### 4. Decay model

Assume exponential decay. A laboratory sample starts at 1240 grams and decreases continuously at 4% per hour. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the time  $t$  when the modeled amount reaches 620 grams. Round to 2 decimal places. Write the model and show your work.

**Model:** \_\_\_\_\_

**Work / substitution:**

---

---

**Answer (hours):** \_\_\_\_\_

### 5. Growth model

Assume exponential growth. A cell culture starts at 400 cells and doubles every 3 hours. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the time  $t$  when the modeled amount reaches 800 cells. Round to 2 decimal places. Write the model and show your work.

**Model:** \_\_\_\_\_

**Work / substitution:**

---

---

**Answer (hours):** \_\_\_\_\_

### 6. Decay model

Assume exponential decay. A laboratory sample starts at 440 grams and has a half-life of 4 days. Let  $t$  be measured in days from the stated origin  $t=0$ . Find the modeled amount at  $t=5$  days. Round to 2 decimal places. Write the model and show your work.

**Model:** \_\_\_\_\_

**Work / substitution:**

---

---

**Answer (grams):** \_\_\_\_\_

# Exponential Growth and Decay

## Worked Answer Key

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

### 1. Growth: culture observations

Assume exponential growth. A cell culture measures 1200 cells at  $t=6$  hours and 2571 cells at  $t=13$  hours. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the modeled amount at  $t=16$  hours. Round the model estimate to the nearest whole cell. Write the model and show your work.

Define variables:  $t$  is time in hours from  $t=0$ ;  $Q(t)$  is the modeled amount in cells.

Derive  $k$ : Using both observations gives  $k=\ln(2571/1200)/7$ .

**Model**

$$Q(t) = 1200 \cdot e^{\frac{\ln\left(\frac{857}{400}\right)}{7}(t-6)}$$

**Derivative law**

$$Q'(t) = \frac{\ln\left(\frac{857}{400}\right)}{7} \cdot Q(t)$$

Calibrate:  $Q(6)=1200$  exactly from the calibrated model;  $Q(13)=2571$  exactly from the calibrated model

**Substitute requested time**

$$Q(16) = 1200 \cdot e^{\frac{\ln\left(\frac{857}{400}\right)}{7} \cdot 10}$$

Approximate answer: 3564 cells

Check: calibration values reproduce the given data without rounding. At  $t=16$  hours,  $Q(t)$  is approximately 3563.8982 cells, compared with 3563.898195893672 cells; this verifies the requested substitution. Final contextual answer: approximately 3564 cells.

# Exponential Growth and Decay

## Worked Answer Key

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

### 2. Decay: sample observations

Assume exponential decay. A laboratory sample measures 1240 grams at  $t=6$  hours and 670 grams at  $t=13$  hours. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the time  $t$  when the modeled amount reaches 469 grams. Round to 2 decimal places. Write the model and show your work.

Define variables:  $t$  is time in hours from  $t=0$ ;  $Q(t)$  is the modeled amount in grams.

Derive  $k$ : Using both observations gives  $k=\ln(670/1240)/7$ .

**Model**

$$Q(t) = 1240 \cdot e^{\frac{\ln\left(\frac{67}{124}\right)}{7}(t-6)}$$

**Derivative law**

$$Q'(t) = \frac{\ln\left(\frac{67}{124}\right)}{7} \cdot Q(t)$$

Calibrate:  $Q(6)=1240$  exactly from the calibrated model;  $Q(13)=670$  exactly from the calibrated model

**Take natural logs**

$$\frac{\ln\left(\frac{67}{124}\right)}{7} \cdot (t-6) = \ln\left(\frac{469}{1240}\right)$$

**Isolate  $t$**

$$t = 6 + \frac{\ln\left(\frac{469}{1240}\right)}{\frac{\ln\left(\frac{67}{124}\right)}{7}}$$

Approximate answer: 17.06 hours

Check: calibration values reproduce the given data without rounding. At  $t=17.055831$  hours,  $Q(t)$  is approximately 469 grams, compared with 469 grams; this verifies the requested substitution. Final contextual answer: approximately 17.06 hours.

# Exponential Growth and Decay

## Worked Answer Key

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

### 3. Growth: culture continuous

Assume exponential growth. A cell culture starts at 1200 cells and increases continuously at 4% per hour. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the modeled amount at  $t=16$  hours. Round the model estimate to the nearest whole cell. Write the model and show your work.

Define variables:  $t$  is time in hours from  $t=0$ ;  $Q(t)$  is the modeled amount in cells.

Derive  $k$ : The stated continuous relative rate gives  $k=4/100$ .

**Model**

$$Q(t) = 1200 \cdot e^{\frac{1}{25} \cdot t}$$

**Derivative law**

$$Q'(t) = \frac{1}{25} \cdot Q(t)$$

Calibrate:  $Q(0)=1200$  exactly from the calibrated model

**Substitute requested time**

$$Q(16) = 1200 \cdot e^{\frac{1}{25} \cdot 16}$$

Approximate answer: 2276 cells

Check: calibration values reproduce the given data without rounding. At  $t=16$  hours,  $Q(t)$  is approximately 2275.7771 cells, compared with 2275.777055165942 cells; this verifies the requested substitution. Final contextual answer: approximately 2276 cells.

# Exponential Growth and Decay

## Worked Answer Key

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

### 4. Decay: sample continuous

Assume exponential decay. A laboratory sample starts at 1240 grams and decreases continuously at 4% per hour. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the time  $t$  when the modeled amount reaches 620 grams. Round to 2 decimal places. Write the model and show your work.

Define variables:  $t$  is time in hours from  $t=0$ ;  $Q(t)$  is the modeled amount in grams.

Derive  $k$ : The stated continuous relative rate gives  $k=-4/100$ .

**Model**

$$Q(t) = 1240 \cdot e^{\frac{-1}{25} \cdot t}$$

**Derivative law**

$$Q'(t) = \frac{-1}{25} \cdot Q(t)$$

Calibrate:  $Q(0)=1240$  exactly from the calibrated model

**Take natural logs**

$$\frac{-1}{25} \cdot t = \ln\left(\frac{1}{2}\right)$$

**Isolate  $t$**

$$t = 0 + \frac{\ln\left(\frac{1}{2}\right)}{\frac{-1}{25}}$$

Approximate answer: 17.33 hours

Check: calibration values reproduce the given data without rounding. At  $t=17.32868$  hours,  $Q(t)$  is approximately 620 grams, compared with 620 grams; this verifies the requested substitution. Final contextual answer: approximately 17.33 hours.

## Exponential Growth and Decay

### Worked Answer Key

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

#### 5. Growth: culture doubling

Assume exponential growth. A cell culture starts at 400 cells and doubles every 3 hours. Let  $t$  be measured in hours from the stated origin  $t=0$ . Find the time  $t$  when the modeled amount reaches 800 cells. Round to 2 decimal places. Write the model and show your work.

Define variables:  $t$  is time in hours from  $t=0$ ;  $Q(t)$  is the modeled amount in cells.

Derive  $k$ : The doubling time relation gives  $k=\ln(2)/3$ .

**Model**

$$Q(t) = 400 \cdot e^{\frac{\ln(2)}{3} \cdot t}$$

**Derivative law**

$$Q'(t) = \frac{\ln(2)}{3} \cdot Q(t)$$

Calibrate:  $Q(0)=400$  exactly from the calibrated model;  $Q(3)=800$  exactly from the calibrated model

**Take natural logs**

$$\frac{\ln(2)}{3} \cdot t = \ln(2)$$

**Isolate  $t$**

$$t = 0 + \frac{\ln(2)}{\frac{\ln(2)}{3}}$$

Approximate answer: 3.00 hours

Check: calibration values reproduce the given data without rounding. At  $t=3$  hours,  $Q(t)$  is approximately 800 cells, compared with 800 cells; this verifies the requested substitution. Final contextual answer: approximately 3.00 hours.

## Exponential Growth and Decay

### Worked Answer Key

Build an exact exponential model, then solve the contextual amount or time question.

Continuous percentages give  $k=r$ ; periodic percentages give  $k=\ln(1+r)$ . Keep exact values until the final approximation.

#### 6. Decay: sample half life

Assume exponential decay. A laboratory sample starts at 440 grams and has a half-life of 4 days. Let  $t$  be measured in days from the stated origin  $t=0$ . Find the modeled amount at  $t=5$  days. Round to 2 decimal places. Write the model and show your work.

Define variables:  $t$  is time in days from  $t=0$ ;  $Q(t)$  is the modeled amount in grams.

Derive  $k$ : The half-life relation gives  $k=\ln(0.5)/4$ .

**Model**

$$Q(t) = 440 \cdot e^{\frac{\ln\left(\frac{1}{2}\right)}{4} \cdot t}$$

**Derivative law**

$$Q'(t) = \frac{\ln\left(\frac{1}{2}\right)}{4} \cdot Q(t)$$

Calibrate:  $Q(0)=440$  exactly from the calibrated model;  $Q(4)=220$  exactly from the calibrated model

**Substitute requested time**

$$Q(5) = 440 \cdot e^{\frac{\ln\left(\frac{1}{2}\right)}{4} \cdot 5}$$

Approximate answer: 185.00 grams

Check: calibration values reproduce the given data without rounding. At  $t=5$  days,  $Q(t)$  is approximately 184.99721 grams, compared with 184.99721135581723 grams; this verifies the requested substitution. Final contextual answer: approximately 185.00 grams.